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PARALLEL IMPLEMENTATION OF HIGHER ORDER NONLINEAR SYSTEM

BY

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Abstract. The present paper aims at developing analogue parallel implementation of a chaotic generator. The proposed generator is a nonlinear system designed starting from an analog second order oscillator and structuring a higher order system by connecting several elementary building blocks in a parallel group included in a larger feedback loop. The designed architecture is intended for several communication applications and its performance is estimated. Using the designed system for biomedical and voice signals transmission is the final desired goal. Dynamic and statistic results are presented in order to justify the efficiency of the proposed implementation.

Keywords: chaos generation; nonlinear dynamics; parallel implementation; analogue systems.

1. Introduction

Nonlinear systems can perform a wide variety of dynamical behaviors such as constant, periodic, quasi-periodic and chaotic. Chaotic systems have

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applications related to noise generation (Grigoraş, 2011), binary random number calculation (Yalcin, 2004; Yang, 2004) and data encryption for secure communication (Grigoraş, 2012). Chaotic modulation can improve encryption capabilities of communication systems by direct application or by combining it with classical algebraic algorithms. In order to have better performance, the chaotic nonlinear system used for encryption must highlight high values of their Lyapunov exponents and high number of parameters.

The present contribution intends to achieve these requirements by developing a high order analogue nonlinear system highlighting a rich behavior. By parallel connection of several low order linear oscillators, the central structure of the desired encryption system is designed. Its nonlinear structure is achieved by including an algebraic nonlinear function in the feedback loop around the high order linear circuit.

The paper structure is as follows: a design section of the analogue nonlinear system is followed by a large set of simulations that confirm the desired chaotic dynamics of the encryption system. The concluding remarks show possible applications of the designed system in secure biomedical and voice signal communication.

2. Analogue System Design

The higher order nonlinear system to be designed is composed of a structure of parallel connection of M second order elementary building blocks, leading to the order:

$$Ord = N + 1 = 2 \cdot M + 1 \quad (1)$$

The resulting block diagram of the designed system in presented in Fig. 1.

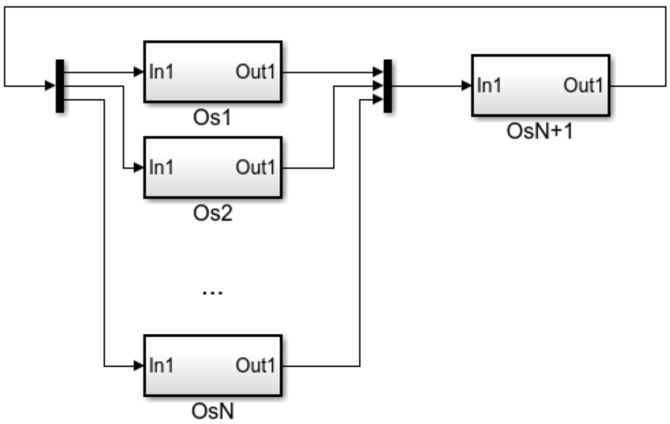


Fig. 1 – Block diagram of the resulting nonlinear system.

The elementary building block of the designed nonlinear system is a second order non-autonomous linear harmonic oscillator, described by the state equations:

$$\begin{cases} \dfrac{\mathrm{d}x_n}{\mathrm{d}t} = a_n \cdot x_n + \omega_n \cdot x_{n+1} + e_n \\ \dfrac{\mathrm{d}x_{n+1}}{\mathrm{d}t} = -\omega_{n+1} \cdot x_n + e_{n+1} \end{cases} \quad n = 1, \ldots, N. \tag{2}$$

The free oscillating frequency of this system is decided by the coefficients ω_n and ω_{n+1}, but the generated signal amplitude cannot be locally designed and will result from the input signals e_n and e_{n+1}, used to close the nonlinear feedback loop, as presented in Fig. 1. For M such oscillators, parallel connected in the nonlinear feedback loop, the resulting $N + 1$ order system is described by the state equations:

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = a_1 \cdot x_1 + \omega_1 \cdot x_2 + b_1 \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = -\omega_2 \cdot x_1 + b_2 \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_3}{\mathrm{d}t} = a_3 \cdot x_3 + \omega_3 \cdot x_4 + b_3 \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_4}{\mathrm{d}t} = -\omega_4 \cdot x_3 + b_4 \cdot x_{N+1} \\ \cdots \\ \dfrac{\mathrm{d}x_{N-1}}{\mathrm{d}t} = a_{N-1} \cdot x_{N-1} + \omega_{N-1} \cdot x_N + b_{N-1} \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_N}{\mathrm{d}t} = -\omega_N \cdot x_{N-1} + b_N \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_{N+1}}{\mathrm{d}t} = \sum_{n=1}^{N} c_n \cdot x_n + f\left(x_1\right) \end{cases} \tag{3}$$

In equation (3), $f(.)$ is the nonlinear algebraic function included in the final system and it can be a comparator nonlinearity (4) or a saturated follower (5):

$$f(x) = sign(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases} \tag{4}$$

$$f(x) = sat(x) = \begin{cases} 1 & x > 1 \\ x & -1 < x < 1 \\ -1 & x < -1 \end{cases} \tag{5}$$

It is important to note that the system described by equation (3) is characterized by a high order, helpful in obtaining a large enough Lyapounov exponent and the large number of coefficients that can be structured in vectors:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \cdots \\ a_N \end{bmatrix} \quad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \cdots \\ b_N \end{bmatrix} \quad \mathbf{c} = \begin{bmatrix} c_1 \\ c_2 \\ \cdots \\ c_N \end{bmatrix} \quad \boldsymbol{\omega} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \cdots \\ \omega_N \end{bmatrix} \tag{6}$$

where:

$$a_{2k} = 0. \tag{7}$$

By denoting the state vector, except the last state variable, x_{N+1} with:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_N \end{bmatrix} \quad \mathbf{y} = \begin{bmatrix} x_2 \\ x_1 \\ x_4 \\ x_3 \\ \vdots \\ x_N \\ x_{N-1} \end{bmatrix} \tag{8}$$

State equations (3) can be written in vector form:

$$\begin{cases} \dfrac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{a} \cdot \mathbf{x} + \boldsymbol{\omega} \cdot \mathbf{y} + \mathbf{b} \cdot x_{N+1} \\ \dfrac{\mathrm{d}x_{N+1}}{\mathrm{d}t} = \mathbf{c}^T \cdot \mathbf{x} + f(\mathrm{x}_1) \end{cases} \tag{9}$$

where products are performed element-by-element from the factor vectors.

The nonlinear dynamics of the designed system and the statistic properties of the generated signals depend on the vectors of coefficient values.

For instance, the dissipative behavior of the system is given by:

$$D = \nabla \mathbf{f} = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \cdots + \frac{\partial f_{N+1}}{\partial x_{N+1}} = \sum_{n=1}^{N} a_n = \sum_{k=0}^{N/2} a_{2k+1} \tag{10}$$

In order to achieve a chaotic behavior, the coefficients structured as vector ‘**a**’ must be chosen to achieve a negative value of their sum:

$$D < 0. \tag{11}$$

3. Simulation Results

In order to perform the desired design in a quantitative way, a parametric analysis can lead us to the parameters values to achieve the followed dynamics. By using bifurcation, we highlighted the fact that the system behaves chaotically for a large range of values of the parameters, with small intervals of periodic behavior. Examples of such diagrams are presented in Fig. 2, for the state variables x_3 and x_5 at the variation of *b*, in the case of equal values of the components of the **b** vector. In these examples, chaotic dynamics is obvious for most values of the coefficient, except values around 0.695, and over 0.705, for which periodic behavior can be viewed.

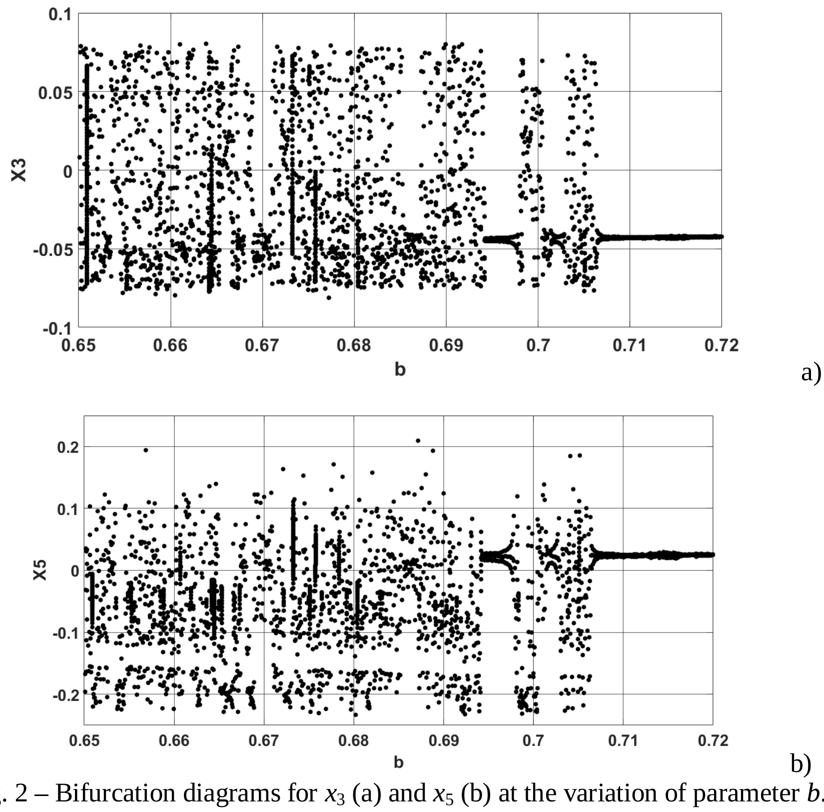
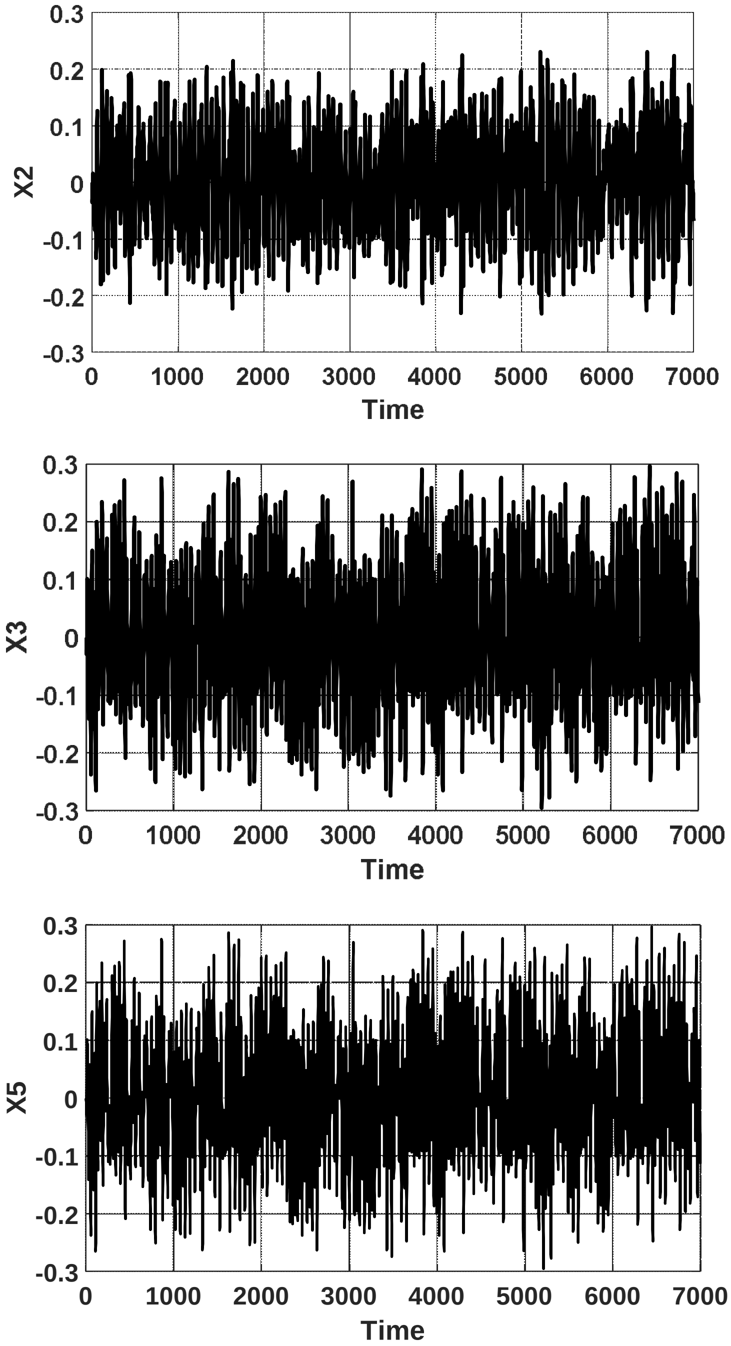


Fig. 2 – Bifurcation diagrams for x_3 (a) and x_5 (b) at the variation of parameter *b*.

For coefficient values suggested by the bifurcation diagrams, time evolution of the state variables is non-periodic and unpredictable, as suggested by the example graphs in Fig. 3.



a)

b)

c)

Fig. 3 – Time evolution of the state variables x_2 (a), x_3 (b) and x_5 (c).

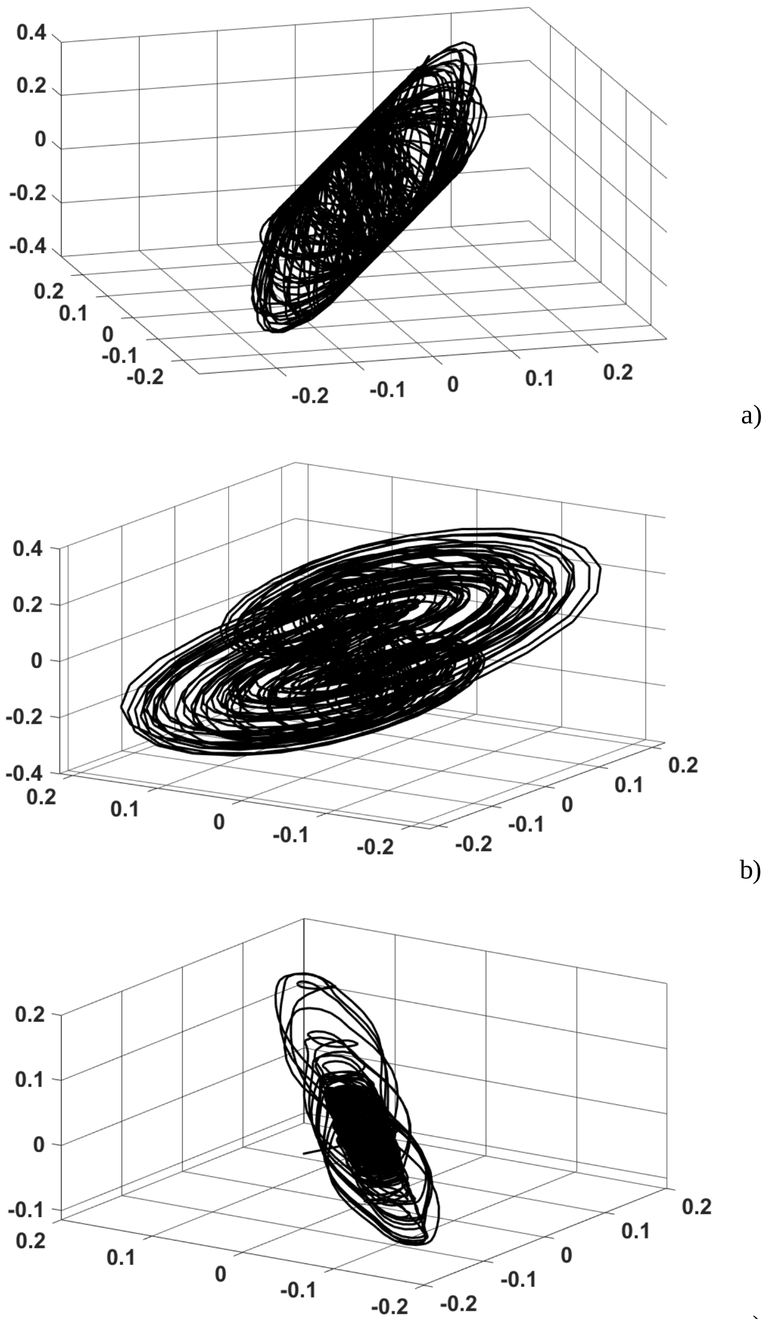


Fig. 4 – 3-D projections of the strange attractor characterizing the dynamics of the system (a – x_1, x_3, x_5) (b – x_1, x_2, x_3) and (c – x_2, x_4, x_5).

Behavioral complexity of the designed system may be noticed from the 3-D projections of the strange attractor, depicted in Fig. 4. The density of these projections highlights ergodicity (topologic transitive behavior) of the designed analog system.

Poincare sections highlight the same unpredictable non-periodic dynamics of the designed system, characteristic for chaotic behavior. The examples depicted in Fig. 5 used the state variable x_1 to decide the sampling moments for the state variables x_2, x_3, and x_2, x_4 respectively.

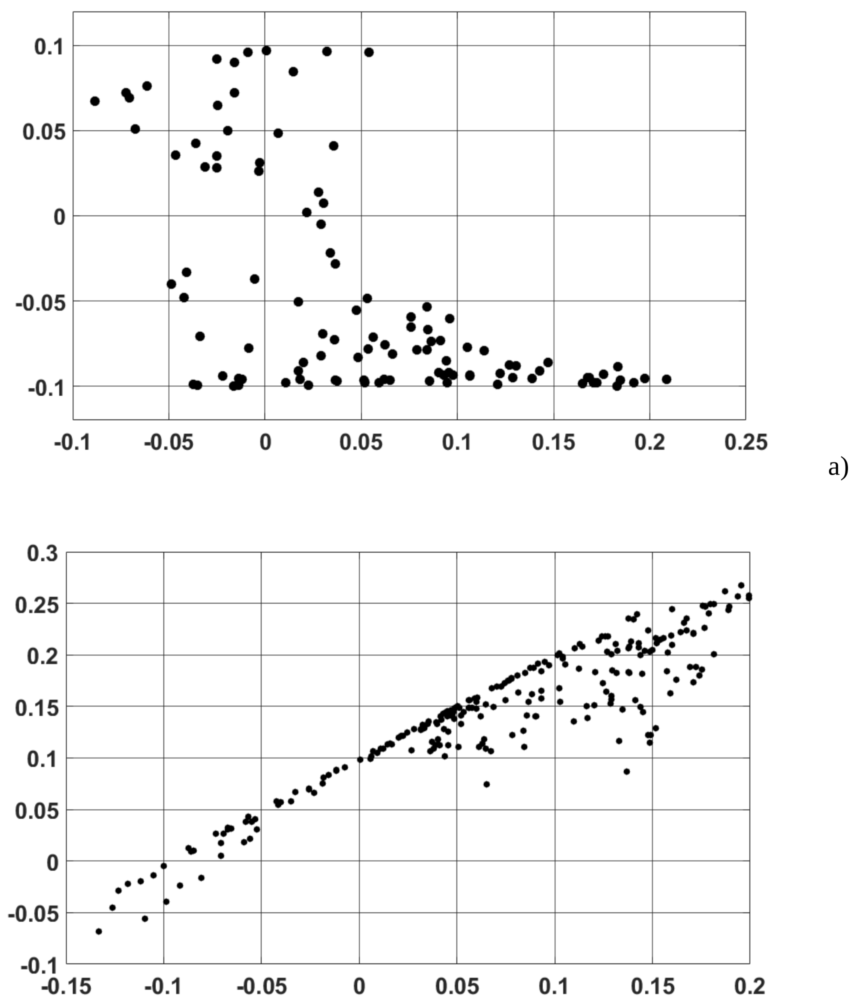
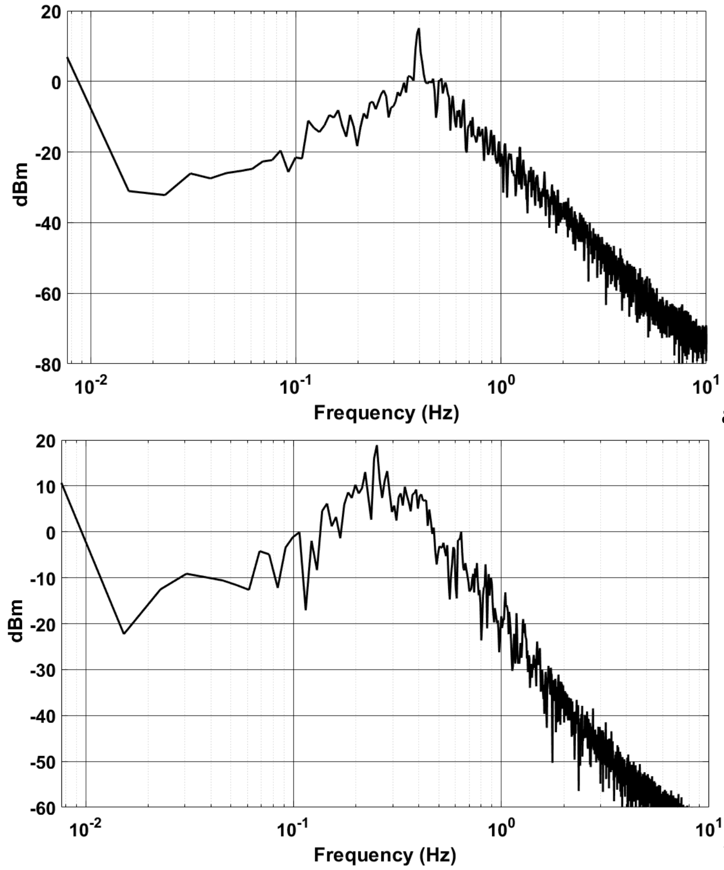


Fig. 5 – Poincare sections for state variables x_2 and x_3 (a) and x_2 and x_4 (*b*).



a)
b)

Fig. 6 – Power spectrum for the state variable x_4, for ω_0 = 2.2 (a) and ω_0 = 1.3 (b).

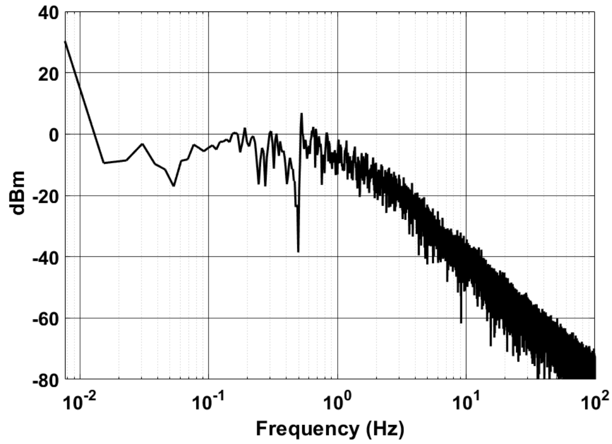


Fig. 7 – Power spectrum for the state variable x_7, for ω_0 = 3.1.

Frequency analysis may also be performed, showing wideband power spectra of the state variables, without dominant frequency lines. Fig. 6 depicts two examples of such power spectra, for the state variable x_4. Both show a small peek at the oscillation frequencies of the second order building blocks. A difference can be noticed for the state variable x_7, for which, at the same frequency a power drop may be seen in Fig. 7.

The dynamic and statistic properties of the designed system highlight high complexity, unpredictable, chaotic behavior. This is useful for modulation and encryption of transmitted signals in increased security communication systems. Use in vocal and biomedical secure storage and/or transmission are some of the possible applications for the proposed nonlinear analogue system.

4. Conclusions

The proposed analogue system shows complex nonlinear dynamics, up to chaotic behavior, for a large variety of parameters. Its high order, large number of state equations parameters and positive Lyapounov exponent ensure the possibility of using it in encrypting biomedical and voice signal for analog communication applications. The parallel structure of the designed system reduces its implementation complexity by facilitating block interconnection and repetitive subsystem design. The proposed system topology also suggests that discrete-time equivalent of the analog system is possible and programmable digital structure is easy to develop.

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IMPLEMENTAREA PARALELĂ A SISTEMELOR NELINIARE DE ORDIN SUPERIOR

(Rezumat)

Această lucrare vizează dezvoltarea unor sisteme analogice pentru generare haotică, implementate parallel. Generatorul propus este un sistem neliniar proiectat pe baza unei structuri paralele de oscilatoare de ordinal do structurate ca un sistem de ordin

superior prin conectarea mai multor blocuri elementare intr-un grup parallel cuprins într-o buclă de reactive mai mare. Arhitectura proiectată vizează mai multe aplicații de comunicații şi se estimează performanțele ei. Aplicarea sistemului proiectat pentru transmiterea de semnale vocale şi biomedicale este scopul final dorit. Rezultatele dinamice şi statistice prezentate justifică eficiența implementării propuse.