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## ANALYZING THE EFFECTS OF SUPPLY VOLTAGE ASYMMETRY ON ELECTROMAGNETIQUE TORQUE OF AN INDUCTION MACHINE

BY

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#### Abstract

When an induction machine is supplied by an asymmetrical voltage system, asymmetrical systems quantities (voltages, currents, fluxes) occur. The same situation happens if the machine is symmetrically supplied but it is asymmetrical either from construction or because of a fault. The current and flux hodographs are elliptical. Finally the electromagnetic torque of the induction machine is affected. Even in steady state the electromagnetic torque is no longer constant. A sinusoidal component (ripple) overlaps its average value. The paper aims to analyze the four components of the electromagnetic torque and to establish a usable analytical relationship that indicates the causes that influence the value of the sinusoidal component of the torque: the asymmetrical supply or the machine failure. Computer simulation in PSpice and also in MATLAB Simulink is used to study the impacts of asymmetrical supply voltages on induction machines.


Keywords: symmetrical components; induction machine; electromagnetic torque ripple.

## 1. Introduction

Most of the studies on the induction machines in the last century have been done considering that the machine operates under a symmetrical three

[^0]phase supply voltage with no distortion. It is designed to work efficiently under sinusoidal and symmetrical supply. Voltage asymmetry can originate from the structural asymmetry of the power supply or unbalance load. When the load is unbalanced, it draws unbalanced current, fluxes and torque harmonics.

This paper refers to an induction machine operating as a motor, so the interest range of rotor speed $\omega_{r}$ is from 0 to $\omega_{1}$, where $\omega_{1}$ is the synchronous speed. The motor is powered from the three phase grid by an asymmetrical voltage system with the dissymmetry coefficient $\varepsilon_{n}=U_{s n} / U_{s p}$, where $U_{s p}$ and $U_{s n}$ are the r.m.s. supply voltages of positive and negative sequence respectively. The motor is symmetrical by construction, each phase having the same parameters.

We have been focused on the electromagnetic torque, which is one of the machine significant quantities. The paper aims to establish a relationship as simple as possible but with minimal errors that allow to determine the influence of both the dissymmetry coefficient of voltage supply and rotor speed on the torque ripple. This is extremely important for the machine diagnose.

## 2. Initial Theoretical Considerations

The mechanical power is the product of the electromagnetic torque and the angular mechanical speed of the rotor. The driving torque is produced by electromagnetic forces acting in the region of the air gap, respectively to the interaction between the currents through windings and the magnetic fluxes. The expression of the torque depending on the stator current and flux was preferred. Using space phasors representation we have:

$$
\begin{equation*}
t_{e}=p\left|\underline{\Psi}_{s} \times \underline{i}_{s}\right| \tag{1}
\end{equation*}
$$

As shown in (Fortescue, 1929) and (Cociu \& Cociu, 2014), the asymmetrical supply leads to positive $(p)$ and negative sequence $(n)$ components of interest quantities:

$$
\begin{gather*}
\underline{\Psi}_{s}=\underline{\Psi}_{s p}+\underline{\Psi}_{s n},  \tag{2}\\
\underline{i}_{s}=\underline{i}_{s p}+\underline{i}_{s n},
\end{gather*}
$$

As a result, the expression of the electromagnetic torque contains four components:

$$
\begin{gather*}
t_{e}=p\left|\left(\underline{\Psi}_{s p}+\underline{\Psi}_{s n}\right) \times\left(\underline{i}_{s p}+\underline{i}_{s n}\right)\right|,  \tag{3}\\
t_{e}=p\left(\underline{\Psi}_{s p} \times \underline{i}_{s p}+\underline{\Psi}_{s n} \times \underline{i}_{s n}+\underline{\Psi}_{s p} \times \underline{i}_{s n}+\underline{\Psi}_{s n} \times \underline{i}_{s p}\right) .
\end{gather*}
$$

We prefer the exponential form of the spatial phasors that highlight modulus (magnitude) and argument (phase). So:

$$
\begin{gather*}
\underline{\Psi}_{s p}=\Psi_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\varphi_{\psi p}\right)},  \tag{4}\\
\Psi_{s p}=\Psi_{s p}\left(\omega_{r}\right) ; \varphi_{\psi p}=\varphi_{\psi p}\left(\omega_{r}\right), \\
\underline{i}_{s p}=I_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\varphi_{i p}\right)},  \tag{5}\\
I_{s p}=I_{s p}\left(\omega_{r}\right) ; \varphi_{i p}=\varphi_{i p}\left(\omega_{r}\right), \\
\underline{\Psi}_{s n}=\Psi_{s n} \mathrm{e}^{\mathrm{j}\left(-\omega_{1} t+\varphi_{\psi n}\right)},  \tag{6}\\
\Psi_{s n}=\Psi_{s p}\left(\omega_{r}\right) \cdot \varepsilon_{n u s} ; \varphi_{\psi n}=\varphi_{\psi n}\left(\omega_{r}\right), \\
I_{s n}=\underline{I}_{s p}\left(\omega_{r}\right) \cdot \varepsilon_{n i s} ; \varphi_{i n}=\varphi_{i n}\left(\omega_{r}\right) .
\end{gather*}
$$

The phasors rotate at synchronous speed in the direct $\left(\mathrm{e}^{\mathrm{j} \omega_{1} t}\right)$ or reverse ( $\mathrm{e}^{-\mathrm{j} \omega_{1} t}$ ) trigonometric direction. The position at the initial moment is determined by the initial phases $\varphi_{i p}, \varphi_{i n}, \varphi_{\psi p}, \varphi_{\psi n}$ considered against the reference voltage $\underline{u}_{s p}$. Fig. 1 presents the relative position of interest spatial phasors.


Fig. 1 - Currents and fluxes spatial phasors.
To confirm the rightness of the currents, fluxes and electromagnetic torque expressions, the machine behavior has been simulated and numerical values have been assigned for the machine parameters. It was considered the
neutral connected. An orthogonal model of an induction machine has been performed for two different machines, rated as follows:

$$
\begin{aligned}
& \text { M1: } \\
& P_{n}=11 \mathrm{kWW} ; \quad U_{11 \mathrm{n}}=400 \mathrm{~V} ; \quad f_{1}=50 \mathrm{~Hz} \\
& R_{s}=0.42 \Omega ; \quad R_{r}^{\prime}=0.42 \Omega ; \quad L_{m}=116 \mathrm{mH} \\
& L_{\sigma s}=3.35 \mathrm{mH} ; \quad L_{\sigma r}^{\prime}=3.35 \mathrm{mH} ; \quad J=75 \mathrm{~g} \cdot \mathrm{~m}^{2} \\
& \mathrm{Y} \text { conn. } ; \quad p=1 ; \quad F_{\alpha}=12 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m} \cdot \mathrm{~s} / \mathrm{rad}
\end{aligned}
$$

M2:

$$
P_{n}=110 \mathrm{~kW} ; \quad U_{1 \mathrm{ln}}=400 \mathrm{~V} ; \quad f_{1}=50 \mathrm{~Hz}
$$

$$
R_{s}=0.03 \Omega ; \quad R_{r}^{\prime}=0.03 \Omega ; \quad L_{m}=12.8 \mathrm{mH} ;
$$

$$
L_{\sigma s}=0.34 \mathrm{mH} ; \quad L_{\sigma r}^{\prime}=0.34 \mathrm{mH} ; \quad J=2 \mathrm{Kg} \cdot \mathrm{~m}^{2}
$$

$$
\text { Y conn } . ; \quad p=1 ; \quad F_{\alpha}=21 \cdot 10^{-3} \mathrm{~N} \cdot \mathrm{~m} \cdot \mathrm{~s} / \mathrm{rad}
$$

The simulation results were based on the $d-q$ model implemented in PSpice (Justus, 1993) but were also confirmed on the three-phase model (Cociu \& Cociu, 2011). In the case of asymmetrical supply corresponding to the machine rated values, the positive and negative sequence phasors used are:

$$
\begin{equation*}
\underline{U}_{s p}=400 / \sqrt{2} ; \quad \underline{U}_{s n}=40 / \sqrt{2} \tag{8}
\end{equation*}
$$

The following $d-q$ components results:

$$
\begin{equation*}
\sqrt{2} U_{s d}=440 \mathrm{~V} \quad ; \quad \sqrt{2} U_{s q}=360 \mathrm{~V} \tag{9}
\end{equation*}
$$

Dissymmetry coefficient of power voltage is:

$$
\begin{equation*}
\varepsilon_{n u s}=\frac{U_{s n}}{U_{s p}}=\frac{40 / \sqrt{2}}{400 / \sqrt{2}}=0.1 \tag{10}
\end{equation*}
$$

## 3. Time Constant Torque Components

There are four different components of the electromagnetic torque. The component due to the interaction between the currents and fluxes systems of positive sequence is:

$$
\begin{gather*}
T_{e p}=p \cdot\left(\underline{\psi}_{s p} \times \underline{i_{s p}}\right)=p \cdot\left(\Psi_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\gamma_{\psi p}\right)} \times I_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\varphi_{i p}\right)}\right)=  \tag{11}\\
=p \cdot \Psi_{s p} \cdot I_{s p} \cdot \sin \left(\varphi_{\psi p}-\varphi_{i p}\right) . \\
T_{e p}=p \cdot \Psi_{s p} \cdot I_{s p} \cdot \sin \left(\varphi_{p}\right), \tag{12}
\end{gather*}
$$

Since $\quad I_{s p}=f\left(\omega_{r}\right), \Psi_{s p}=f\left(\omega_{r}\right)$ and $\sin \left(\varphi_{\psi p}-\varphi_{i p}\right)=f\left(\omega_{r}\right)$ it results $T_{e p}=T_{e p}\left(\omega_{r}\right)$.

The component due to the interaction between the currents and fluxes systems of negative sequence is:

$$
\begin{gather*}
T_{e n}=p \cdot\left(\underline{\Psi}_{s n} \times \underline{i}_{s n}\right)=p \cdot\left(\Psi_{s n} \mathrm{e}^{\mathrm{j}\left(-\omega_{1} t+\gamma_{n}\right)} \times I_{s n} \mathrm{e}^{\mathrm{j}\left(-\omega_{1} t+\gamma_{n}+\varphi_{n}\right)}\right)= \\
=-p \cdot \Psi_{s n} \cdot I_{s n} \cdot \sin \left(\varphi_{\psi n}-\varphi_{i n}\right),  \tag{13}\\
T_{e n}=-p \cdot \Psi_{s n} \cdot I_{s n} \cdot \sin \left(\varphi_{n}\right) .
\end{gather*}
$$

Since $\quad I_{s n}=f\left(\omega_{r}\right), \Psi_{s n}=f\left(\omega_{r}\right)$ and $\sin \left(\varphi_{\psi n}-\varphi_{i n}\right)=f\left(\omega_{r}\right)$ it also results $T_{e n}=T_{e n}\left(\omega_{r}\right)$. Using the dissymmetry coefficient definition:

$$
\begin{equation*}
\varepsilon_{n u s}=\frac{U_{s n}}{U_{s p}}, \quad \varepsilon_{n i s}=\frac{I_{s n}}{I_{s p}}, \quad \varepsilon_{n \psi s}=\frac{\Psi_{s n}}{\Psi_{s p}} \tag{14}
\end{equation*}
$$

it is possible to express the torque component depending on the current and flux of negative sequence:

$$
\begin{equation*}
T_{e n}=-p \cdot \varepsilon_{n \psi s} \cdot \varepsilon_{n i s} \Psi_{s p} \cdot I_{s p} \cdot \sin \left(\varphi_{n}\right) \tag{15}
\end{equation*}
$$

The torque components $T_{e p}$ and $T_{e n}$ are constant in time but their values depend on the rotor speed. The expressions of the two components can be obtained with high accuracy using the exact classic expression of the torque in simplified form:

$$
\begin{equation*}
T_{e}=\frac{2 M_{b}\left(1+2 \lambda s_{b}\right)}{\frac{s}{s_{b}}+\frac{s_{b}}{s}+2 \lambda s_{b}}, \tag{16}
\end{equation*}
$$

where: $s$ is the slip, $\lambda=R_{s} /\left(c R_{r}\right), T_{b}$ is the breakdown torque :

$$
\begin{equation*}
T_{b}=\frac{3 p U_{s}^{2}}{2 \omega_{1}\left(R_{s}+\sqrt{R_{s}^{2}+\left(X_{\sigma s}+c X_{\sigma r}\right)^{2}}\right)} \tag{17}
\end{equation*}
$$

and $s_{b}$ is the breakdown slip :

$$
\begin{equation*}
s_{b}=\frac{c R_{r}}{\sqrt{R_{s}^{2}+\left(X_{\sigma s}+c X_{\sigma r}\right)^{2}}} . \tag{18}
\end{equation*}
$$

The torque component $T_{e p}$ expression is the same as eq. (16). The torque component $T_{e n}$ is obtained for $\omega_{r}^{-}=-\omega_{r}<0$, so $1<s^{-}=2-s<2$. Then $T_{e n}$ results as:

$$
\begin{equation*}
T_{e n}=\frac{2 M_{b}\left(1+2 \lambda s_{b}\right)}{\frac{2-s}{s_{b}}+\frac{s_{b}}{2-s}+2 \lambda s_{b}} . \tag{19}
\end{equation*}
$$

For the two numerical examples we obtain:

$$
\begin{aligned}
& \text { M1 } c=1.044, \lambda=0.958, s_{b}=0.2, T_{b}=93.5 \mathrm{Nm} \\
& \text { M2 } c=1.040, \lambda=0.962, s_{b}=0.14, T_{b}=980 \mathrm{Nm} .
\end{aligned}
$$

Fig. 2 shows the variations of the torque versus the rotor speed for M2 example. The $T_{e p}$ component due to the positive sequence components of current and flux has a specific variation which is known in literature. This is due to the variation of the $I_{s p}$ current and also to the phase $\varphi_{\psi p}-\varphi_{i p}$ and especially to the phase $\varphi_{i p}$. The $T_{e n}$ component due to the negative sequence components of current and flux corresponds to the braking regime. Its small variation in the entire range of rotor speed is particularly due to the phase $\varphi_{\psi n}-\varphi_{i n}$.


Fig. 2 - Torque versus speed.

The electromagnetic torque value is positive in both cases because it is of the rotating magnetic field sense. If the torque relates to the direction of the positive field component then $T_{e n}$ has negative values as shown in Fig. 3. The
representation has been made considering the same nominal supply voltage for both components. In fact the negative sequence voltage supply is lower and the corresponding torque results much lower.


Fig. 3 - Constant torque components versus speed.

## 4. Sinusoidal Torque Components

For the other two components obtained from the current - flux interaction of different sequences it results:

$$
\begin{gather*}
t_{e p n}=p\left(\underline{\Psi}_{s p} \times \underline{i}_{s n}\right)=p\left(\Psi_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\varphi_{\psi p}\right)} \times I_{s n} \mathrm{e}^{\mathrm{j}\left(-\omega_{1} t+\varphi_{i n}\right)}\right)=  \tag{20}\\
=\Psi_{s p} \cdot I_{s n} \cdot \sin \left(-\omega_{1} t+\varphi_{i n}-\omega_{1} t+\varphi_{\psi p}\right),
\end{gather*}
$$

and according to (Cociu \& Cociu, 2017):

$$
\begin{gather*}
t_{e p n}=-\Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n i s} \cdot \sin \left(2 \omega_{1} t-\varphi_{i n}-\varphi_{\psi p}\right),  \tag{21}\\
t_{e n p}=p\left(\underline{\Psi}_{s n} \times \underline{i}_{s p}\right)=p\left(\Psi_{s n} \mathrm{e}^{\mathrm{j}\left(-\omega_{1} t+\varphi_{\psi n}\right)} \times I_{s p} \mathrm{e}^{\mathrm{j}\left(\omega_{1} t-\varphi_{i p}\right)}\right)=  \tag{22}\\
=p \Psi_{s n} \cdot I_{s p} \cdot \sin \left(\omega_{1} t-\varphi_{i p}+\omega_{1} t-\varphi_{\psi n}\right),
\end{gather*}
$$

and according to (Cociu \& Cociu, 2018):

$$
\begin{equation*}
t_{e n p}=p \Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n u s} \cdot \sin \left(2 \omega_{1} t-\varphi_{i p}-\varphi_{\psi n}\right) \tag{23}
\end{equation*}
$$

To determine the influence of angular rotor speed on the interest quantities, a quasi-stationary operating is considered. The rotor speed increases
at constant slope. Within a 2 s range the rotor speed increases slowly from 0 to $\gamma_{1}=314 \mathrm{rad} / \mathrm{s}$ that implies the quasi steady state. In previously papers the value of $314 / 2 \mathrm{rad} / \mathrm{s}^{2}$ slope resulted to be a reference value.


Fig. 4 - First sinusoidal torque component versus speed: $a-\mathrm{M} 1, b-\mathrm{M} 2$.
In order to verify the theoretical results by PSpice simulation the mathematical expression as a time function is required. PSpice cannot use the mathematical language to describe spatial phasors. Therefore eqs. (20) and (22) are written as:

$$
\begin{gather*}
t_{e p n}=p\left[\left(\Psi_{s p d}+\mathrm{j} \cdot \Psi_{s p q}\right) \times\left(i_{s n d}+\mathrm{j} \cdot i_{s n q}\right)\right]=p\left(\mathrm{j} \cdot \Psi_{s p q} \times i_{s n d}+\Psi_{s p d} \times \mathrm{j} \cdot i_{s n q}\right)=  \tag{24}\\
=p\left[\Psi_{s p d}(t) \cdot i_{s n q}(t)-\Psi_{s p q}(t) \cdot i_{s n d}(t)\right],
\end{gather*}
$$

$$
\begin{gather*}
t_{e n p}=p\left[\left(\Psi_{s n d}+\mathrm{j} \cdot \Psi_{s n q}\right) \times\left(i_{s p d}+\mathrm{j} \cdot i_{s p q}\right)\right]=p\left(\mathrm{j} \cdot \Psi_{s n q} \times i_{s p d}+\Psi_{s n d} \times \mathrm{j} \cdot i_{s p q}\right)=  \tag{25}\\
=p\left[\Psi_{s n d}(t) \cdot i_{s p q}(t)-\Psi_{s n q}(t) \cdot i_{s p d}(t)\right] .
\end{gather*}
$$

In Figs. 4 and 5 are presented the simulation results for $t_{e n p}$ and $t_{e p n}$ respectively. We used two scales - time and rotor speed - on the horizontal axis to highlight the linear variation with time of the rotor speed.


Fig. 5 -Second alternative torque component versus speed: $a-\mathrm{M} 1, b-\mathrm{M} 2$.
The amplitude of the sinusoidal component $t_{e p n}$ (Fig.4) is relatively constant having the aspect of the variation of the stator flux versus rotor speed. The amplitude of the sinusoidal component $t_{\text {enp }}$ (Fig. 5) is not constant having the aspect of the variation of the stator current versus rotor speed.

This two components are sinusoidal of same frequency $2 \gamma_{1}$. So their cumulative effect is obtained by addition:

$$
\begin{gather*}
t_{e \sim}=t_{e p n}+t_{e n p}=-p \Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n i s} \cdot \sin \left(2 \omega_{1} t-\varphi_{i n}-\varphi_{\psi p}\right)+ \\
\quad+p \Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n u s} \cdot \sin \left(2 \omega_{1} t-\varphi_{i p}-\varphi_{\psi n}\right)=  \tag{26}\\
=T_{e p n} \cdot \sin \left(2 \omega_{1} t-\varphi_{i p}-\varphi_{\psi n}\right)-T_{e n p} \cdot \sin \left(2 \omega_{1} t-\varphi_{i n}-\varphi_{\psi p}\right) .
\end{gather*}
$$

The variation of their initial phases versus the rotor speed has been determined by simulation. The results are presented in Fig. 6. Relatively small variations for $\varphi_{i n}, \varphi_{\psi p}, \varphi_{\psi n}$, but major changes for $\varphi_{i p}$ are noticed. These results can be used to simplify the torque expression.


Fig. 6 - Initial phases versus speed.
In order to add the two sinusoidal components of same frequency, the simplified phasor representation is used, as shown in Fig. 7. The addition result is also a sinusoidal signal of the same frequency:

$$
\begin{equation*}
t_{e \sim}=t_{e p n}+t_{e n p}=T_{e \sim} \sin \left(2 \omega_{1}+\varphi_{\sim}\right) \tag{27}
\end{equation*}
$$

where: $T_{e \sim}$ is the magnitude and $\varphi_{\sim}$ the initial phase of the resulting signal:

$$
\begin{equation*}
T_{e \sim}=\sqrt{T_{e p n}^{2}+T_{e n p}^{2}-2 T_{e p n} T_{e p n} \cos \left(\varphi_{i p}+\varphi_{\psi n}-\varphi_{i n}-\varphi_{\psi p}\right)} . \tag{28}
\end{equation*}
$$

The two sinusoidal components are added according to vectorial rules. Analyzing the graphical representations in both Figs. 7 and 6 it can be seen that in almost all the range of rotor speed the two phasors are opposite, so it results, in a first approximation, the operation of subtraction of the modules:

$$
\begin{equation*}
T_{e \sim}^{*} \approx T_{e p n}-T_{e n p} \tag{29}
\end{equation*}
$$



Fig. 7 - Sinusoidal components diagram for different rotor speed, [rad/s].
Therefore:

$$
\begin{equation*}
T_{e \sim}^{*} \approx p \Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n i s}-p \Psi_{s p} \cdot I_{s p} \cdot \varepsilon_{n u s}=p \Psi_{s p}\left(I_{s p} \cdot \varepsilon_{n i s}-I_{s p} \cdot \varepsilon_{n u s}\right) \tag{30}
\end{equation*}
$$

According to (Cociu \& Cociu, 2017) it results:

$$
\begin{equation*}
I_{s p} \cdot \varepsilon_{n i s}=I_{s n}=I_{s z} \cdot \varepsilon_{n u s} \tag{31}
\end{equation*}
$$

where $I_{s z}$ is the short circuit current respectively starting current.

$$
\begin{equation*}
T_{e \sim}^{*} \approx p \cdot \Psi_{s p} \cdot \varepsilon_{n u s}\left(I_{s z}-I_{s p}\right) \tag{32}
\end{equation*}
$$

Since $p, I_{s z}, \varepsilon_{\text {nus }}=\mathrm{const}$. and also $\Psi_{s p} \approx \mathrm{const}$. (Cociu \& Cociu, 2017), (Cociu \& Cociu, 2018), it results $T_{e \sim}=f\left(\omega_{r}\right)$ because the stator current variation $I_{s p}=f\left(\omega_{r}\right)$.

In these conditions, the expression of the positive sequence component of the stator current becomes:

$$
\begin{equation*}
I_{s p}=\frac{U_{s p}}{Z}=U_{s p}\left|\frac{1+\underline{Z}_{r}^{\prime} / \underline{Z}_{m}}{\mid \underline{Z}_{s}+\underline{c} \cdot \underline{Z}_{r}^{\prime}}\right|, \tag{33}
\end{equation*}
$$

where: $I_{s p}$ depends on the rotor speed through $\underline{Z}_{r}^{\prime}$, respectively $R_{r}^{\prime} / s$.
The alternative components overlap the constant values components. Fig. $8 a$ and Fig. $9 a$ shows the variation of the electromagnetic torque in the quasi-steady state. It can be noticed that the torque ripple changes with the rotor speed.

Fig. 8 b and Fig 9 b show only the time variation of the sinusoidal component that is the torque ripple. The amplitude is zero at starting and increases constantly with the rotor angular speed.

The approximate expression of the torque ripple amplitude is given by eq. (32). Important differences between the approximate and real ripple are noticed. This is because the ripple is obtained by subtracting two alternative components of comparable amplitude (29). Hence the phase difference between signals plays a decisive role. Namely $t_{e p n}$ component phase is $\left(-\varphi_{i n}-\varphi_{\psi p}\right)$
whereas $t_{e n p}$ component phase is $\left(-\varphi_{i p}-\varphi_{\psi n}\right)$. These phases get equals values for $\gamma_{r}=0$ (Fig. 6) but change significantly (especially $\varphi_{i p}$ ) with the rotor speed increase.


Fig. 8 -Torque versus speed in quasi-steady state operating - M1: $a$ - total torque; $b$ - ripple torque.

Analyzing the phase values leads to extremely intricate and difficult to use expressions. (32) is simple but far from reality.

A careful and detailed analysis of the causes (impedances) that generate these phase differences allows to found a simple and easy to use expression which can correct the errors (Cociu \& Cociu, 2019):

$$
\begin{equation*}
k_{c o r}=s \cdot \frac{I_{s z}}{I_{s 0}}+1-s \tag{34}
\end{equation*}
$$

Hence the ripple corrected expression is (Cociu \& Cociu, 2019):

$$
\begin{equation*}
T_{e \sim}^{c o r} \approx p \cdot \Psi_{s p} \cdot \varepsilon_{n u s}\left(I_{s z}-I_{s p}\right)\left(s \cdot \frac{I_{s z}}{I_{s 0}}+1-s\right) . \tag{35}
\end{equation*}
$$



Fig. 9 - Torque versus speed in quasi-steady state operating - M2: a - total torque; b - ripple torque

## 5. Conclusions

When an induction machine is supplied by an asymmetrical voltage system, asymmetrical systems of interest quantities (voltages, currents, fluxes) occur. The same situation happens when symmetrically supply but the induction machine is asymmetrical either from construction or because of a fault.

Performance of induction motor powered by asymmetrical and/or nonsinusoidal supply voltages changes. Increasing both the harmonic distortions level and the voltage asymmetry in power grid can affect the efficiency and the reliability of the induction motors operation (Kostic \& Nikolic, 2010).

The supply voltage asymmetry causes that the current and field get direct and reverse components. It results four components of the electromagnetic torque: two time constant components and two sinusoidal components of $2 \omega_{1}$ angular frequency as the torque ripple.

The first sinusoidal component is of approximate constant amplitude, not depending of rotor speed, due to $I_{s n}=$ const. and $\Psi_{s p} \approx$ const. The amplitude of the second component changes with the rotor speed because of the stator current $I_{s p}=f\left(\omega_{r}\right)$. The sinusoidal components (ripple) overlap the constant values components. The ripple amplitude is proportional to both the direct component of the stator flux $\Psi_{s p}$ and supply voltage dissymmetry coefficient $\varepsilon_{n u s}$.

The addition of ripple components gives a sinusoidal signal of $2 \omega_{1}$ angular frequency. The ripple is obtained by the difference of two alternative components of comparable amplitudes. Therefore the phase difference between the signals plays an important role.

In this paper the mathematical expression of the torque ripple has been established in order to found the factors that determine the torque ripple value. The torque ripple resulted to be proportional with the dissymmetry coefficient and dependent of the rotor speed.

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## ANALIZA EFECTULUI ALIMENTĂRII ASIMETRICE ASUPRA CUPLULUI ELECTROMAGNETIC AL MAŞINII ASINCRONE


#### Abstract

(Rezumat) Când o maşină asincronă este alimentată cu un sistem trifazat asimetric de tensiuni, apar sisteme asimetrice de mărimi de interes (tensiuni, curenți, fluxuri). Aceeaaşi situaație se întâmplă în cazul alimentării simetrice, dar maşina asincronă este asimetrică fie din construcție, fie din cauza unui defect. Hodograful curentului şi fluxului magnetic devine eliptic. În final, cuplul electromagnetic al maşinii asincrone este afectat. Chiar şi în regim permanent cuplul electromagnetic nu mai este constant. O componentă sinusoidală (riplu) se suprapune peste valoarea medie. Lucrarea îşi propune să analizeze cele patru componente ale cuplului electromagnetic şi să stabilească o relație analitică utilizabilă care indică cauzele care influențează valoarea componentei sinusoidale a cuplului: alimentarea asimetrică sau defectul din maşină. Simularea computerizată în PSpice, dar şi în MATLAB Simulink este utilizată pentru a studia efectul tensiunilor de alimentare asimetrice asupra maşinii asincrone.


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