BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Volumul 66 (70), Numărul 2, 2020
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

CONSIDERATIONS ABOUT CURRENT-SPEED EXPRESSIONS OF THE INDUCTION MACHINE

BY

VOINEA-RADU COCIU[*] and LIVIA COCIU

“Gheorghe Asachi” Technical University of Iaşi,
Faculty of Electrical Engineering, Energetics and Applied Informatics

Received: June 22, 2020
Accepted for publication: December 8, 2020

Abstract. The paper presents an analysis of the classical expressions of the induction machine currents. The simplifications used are highlighted and the errors caused by these simplifications are evaluated. The absolute errors are calculated on a practical application and compared to those obtained by simulation, based on the full two-phase or three-phase model. Comments are made on the values of absolute and relative errors. Finally, starting from the classical (simplified) expression of the electromagnetic torque, a new expression of the rotor current is proposed, which highlights first of all the variation of the current versus the rotor speed (slip).

Keywords: induction machine; current expression; rotor current versus speed; PSpice.

1. Introduction

Induction motors are the most used motors in industrial control systems, as well as in main powered home appliances. They have simple and rugged design, low-cost, low maintenance and direct connection to the grid. They are extensively used in both fixed-speed and variable-speed services. Various types of induction motors suitable for different applications are available in the

[*]Corresponding author; *e-mail*: cociuvr@tuiasi.ro

market. Although induction motors are easier to design than dc motors, the speed and the torque control in various types of induction motors require a greater understanding of their characteristics.

The efficient use of an induction motor requires the prior knowledge of the basic torque-speed and current-speed characteristics, which must be matched with the mechanical characteristics of the load and of the electrical supply.

A good understanding of the characteristics is extremely important to properly use the induction motors in different applications, so that the load torque and the motor torque are relatively adequate (Rockwell Automation, 1996; Agamloh *et al*., 2016).

To describe the behavior of the induction machines mathematical models with distributed or concentrated parameters have been developed. The *d*-*q* (orthogonal) model is a natural model for single-phase and two-phase induction machines. For the three-phase induction machines the three-phase model in phase coordinates is more suitable. Most times however, the *d*-*q* (two-phase) model is used in the study of the three-phase induction machine operating either as a motor or a generator.

2. The Currents Expression

The behavior of the asynchronous machine in classical study is described by means of specific equations. In case of the machine structural symmetry we consider the induction motor connected to the three-phase grid as a three-phase balanced load. Under this condition a three phase induction machine can be analyzed using the equivalent circuit of a single phase (per-phase equivalent circuit) (Gheorghiu and Fransua, 1971; Lyshevsky, 1999), where the per-phase currents expression are derived. Fig. 1 shows the electrical diagram of one phase in which the machine constructive parameters are specified, respectively the resistors and inductors. It should be noted that this scheme is valid only in steady state, that is at constant speed $\omega_r = const.$

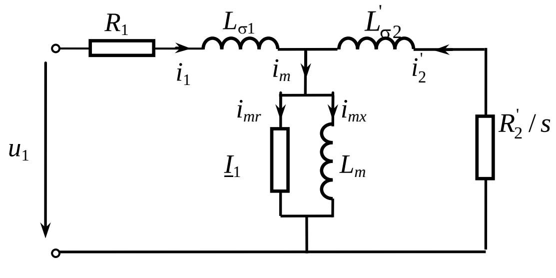

Fig. 1 – Per-phase equivalent circuit of an induction motor.

To confirm the rightness of analytical results the machine behavior has been simulated and numerical values have been assigned for the machine parameters. It is considered the neutral connected. *d-q* model of an induction machine rated as follows has been performed:

$$P_n = 5.5\,\text{kW},\quad f_1 = 50\,Hz,\qquad U_1 = 400\,\text{V},$$
$$R_1 = 0.7\,\Omega;\qquad L_{\sigma 1} = 6\,\text{mH};$$
$$R_r^{'} = 0.67\,\Omega,\quad L_{\sigma 2}^{'} = 5.7\,\text{mH};$$
$$L_m = 90\,\text{mH};\quad R_m = 1.3\,\text{k}\Omega\quad p = 1;\quad \text{Y}\,conn.;$$

A careful analysis reveals that the impedance value on each phase depends on the angular speed (or slip): $\underline{Z} = \underline{Z}(\omega_r) = \underline{Z}(s)$. The impedances are therefore parametric elements and the generalized balanced load theory cannot be used (Popa 2010; Cociu and Cociu 2014).

We use the following notation:

$$\begin{aligned} \underline{Z}_1 &= R_1 + \text{j}X_{\sigma 1} \\ \underline{Z}_m &= \text{j}X_m // R_m \\ \underline{Z}_2^{'} &= R_2^{'}/s + \text{j}X_{\sigma 2}^{'} \end{aligned} \tag{1}$$

so the per-phase equivalent circuit is as shown in Fig. 2:

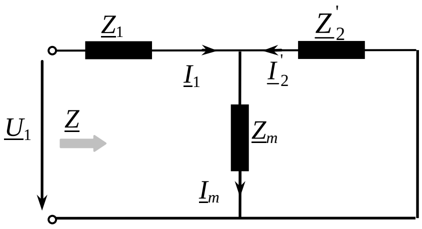

Fig. 2 – Per-phase equivalent circuit.

It results from Fig.2:

$$\underline{Z} = \underline{Z}_1 + \underline{Z}_m \cdot \underline{Z}_2^{'}/(\underline{Z}_m + \underline{Z}_2^{'}) = \frac{\underline{Z}_1 + \underline{Z}_r^{'}(1 + \underline{Z}_1/\underline{Z}_m)}{1 + \underline{Z}_r^{'}/\underline{Z}_m} \tag{2}$$

$$\underline{Z}_m = R_m // \text{j}X_m = \frac{\text{j}X_m R_m}{R_m + \text{j}X_m} = \text{j}X_m \frac{R_m}{R_m + \text{j}X_m} \approx \text{j}X_m \tag{3}$$

We use the classical notation:

$$\underline{c}_1 = 1 + \underline{Z}_1/\underline{Z}_m = 1 + \frac{R_1 + \text{j}X_{\sigma 1}}{\text{R}_\text{m}//\text{jX}_\text{m}} = 1 + \frac{X_{\sigma 1}}{\text{X}_\text{m}} + \frac{R_1}{R_m} + \text{j}\left(\frac{X_{\sigma 1}}{R_m} - \frac{\text{R}_1}{\text{X}_\text{m}}\right) = c_1 e^{\text{j}\gamma} \tag{4}$$

$\underline{c}_1$ is a complex number slightly greater than unit with a small negative angle γ. For the numerical example we considered it results:

$$\underline{c}_1 = 1.066 - j0.024 = 1.0669 e^{j(-0.024)} \tag{5}$$

The typical approximation is:

$$\underline{c}_1 = 1 + \underline{Z}_1 / \underline{Z}_m \approx 1 + X_{\sigma 1} / X_m = c_1 \tag{6}$$

Thus, for the input impedance (at the stator terminals) we get:

$$\underline{Z} = \frac{\underline{Z}_1 + c_1 \cdot \underline{Z}_2^{'}}{1 + \underline{Z}_2^{'} / \underline{Z}_m} \tag{7}$$

The stator current results as:

$$\underline{I}_1 = \frac{\underline{U}_1}{\underline{Z}} = \underline{U}_1 \frac{1 + \underline{Z}_2^{'} / \underline{Z}_m}{\underline{Z}_1 + c_1 \cdot \underline{Z}_2^{'}} \tag{8}$$

The impedances $\underline{Z}_2^{'}$ and $\underline{Z}_m$ form a current divider (Fig. 2) therefore the other two currents of interest – the rotor current and the magnetizing current can be expressed as follows (depending on the stator current):

$$\underline{I}_2^{'} = -\underline{I}_1 \frac{\underline{Z}_m}{\underline{Z}_m + \underline{Z}_2^{'}} = -\frac{\underline{U}_1}{\underline{Z}_1 + c_1 \cdot \underline{Z}_2^{'}} \tag{9}$$

$$\underline{I}_m = \underline{I}_1 \frac{\underline{Z}_2^{'}}{\underline{Z}_m} = \frac{\underline{U}_1}{\underline{Z}_1 + c_1 \cdot \underline{Z}_2^{'}} \frac{\underline{Z}_2^{'}}{\underline{Z}_m} \tag{10}$$

Using the simplification (3) the magnitude of the three currents can be expressed as:

$$I_1 = \frac{U_1}{\omega L_m} \sqrt{\frac{\left(R_2^{'} / s\right)^2 + \omega^2 \left(L_m + L_{\sigma 2}^{'}\right)^2}{\left(R_1 + c_1 R_2^{'} / s\right)^2 + \omega^2 \left(L_{\sigma 1} + c_1 L_{\sigma 2}^{'}\right)^2}} \tag{11}$$

$$I_2^{'} = \frac{U_1}{\sqrt{\left(R_1 + c_1 R_2^{'} / s\right)^2 + \omega^2 \left(L_{\sigma 1} + c_1 L_{\sigma 2}^{'}\right)^2}} \tag{12}$$

$$I_m = \frac{U_1}{\omega L_m} \sqrt{\frac{\left(R_2^{'} / s\right)^2 + \omega^2 L_{\sigma 2}^{'2}}{\left(R_1 + c_1 R_2^{'} / s\right)^2 + \omega^2 \left(L_{\sigma 1} + c_1 L_{\sigma 2}^{'}\right)^2}} \tag{13}$$

Although two approximations – (3) and (6) – were used, the current expressions are quite complicated but very close to reality. Next we aim to evaluate these errors.

3. Errors in Classical Current Expressions

To validate the currents expressions PSpice simulation has to be carried out (Justus, 1993). The *d*-*q* model developed on equivalent circuit in Fig. 1 (Cociu and Cociu, 2005) has been used. But the results have been confirmed on the three-phase model. The same values of the machine constructive parameters have been used together with the relationship $L_0 = (2/3)L_m$ (Cociu and Cociu, 2011).

a) Stator current

In the particular case of running at synchronous speed, that is zero slip, we obtain:

$$I_2^{'}\Big|_{s=0} = 0 \tag{14}$$

$$I_0 = I_1\Big|_{s=0} = I_m\Big|_{s=0} = I_m = \frac{U_1}{\omega c_1 L_m} = \frac{U_1}{X_m + X_{\sigma 1}} \tag{15}$$

The exact value of the no-load current is:

$$I_0^* = \frac{U_1}{\sqrt{R_1^2 + \left(X_m + X_{\sigma 1}\right)^2}} \tag{16}$$

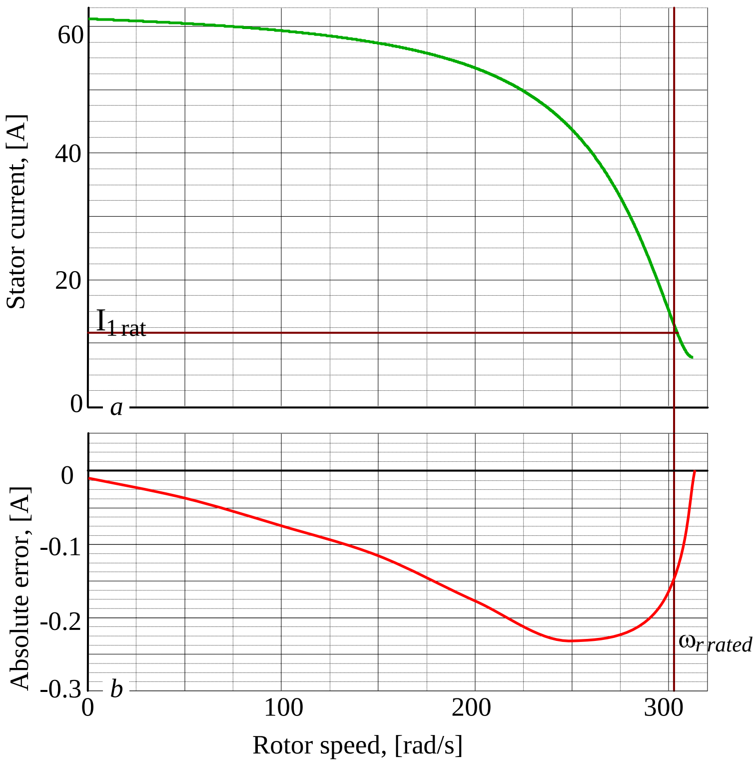

Fig. 3 – Stator current versus speed: *a* – current value; *b* – absolute error.

Because $X_m + X_{\sigma 1} >> R_1$ the error is reduced in this case and it can be neglected. In the numerical example we considered, it results $\varepsilon = \Delta I_0 / I_0^* = 0.027\%$. In Fig. 3 are presented the values of the stator current obtained by using (11). The errors compared to the exact values obtained by simulation have been also determined. These errors are caused by the approximations (3) and (6). The absolute error has very small values at both starting (s = 1) and synchronism (s = 0). Higher values are obtained around the break-down slip s_k. The absolute error reaches the value $\Delta I_{1\max} = 0.24A$ and the relative error $\varepsilon_{1\max} = \Delta I_1 / I_{1r} = 1.9\%$. In normal load operation the relative error is $\varepsilon = \Delta I_1 / I_{1r} < 1\%$.

b) Rotor current

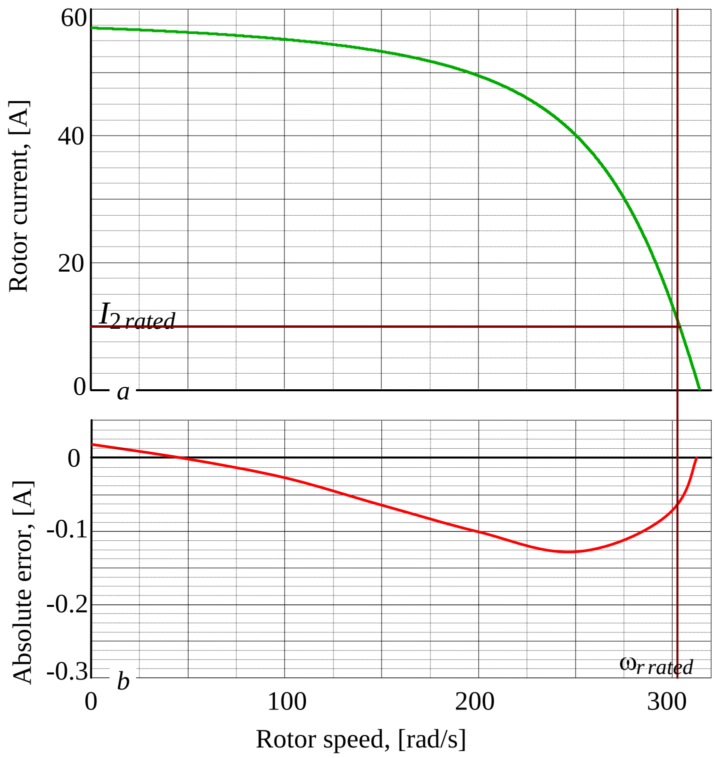

Fig. 4 – Rotor current versus speed: *a* – current value; *b* – absolute error.

In Fig. 4 are presented the values of the rotor current obtained by using (12) and the errors compared to the exact values obtained by simulation. The errors are due firstly to approximation (6) and secondly to approximation (3).

As in the previous case, very small errors are obtained at starting (s = 1) and at synchronism (s = 0). Larger errors are also obtained close to the break-down slip s_k, but the maximum absolute error $\Delta I'_{2\max} = 0.13A$ and the relative error $\varepsilon_{2\max} = \Delta I'_2 / I'_{2r} = 1\%$ are lower than in the previous case. In the regular load operation the relative error is $\varepsilon_2 = \Delta I'_2 / I'_{2r} < 0.5\%$.

c) Magnetizing current

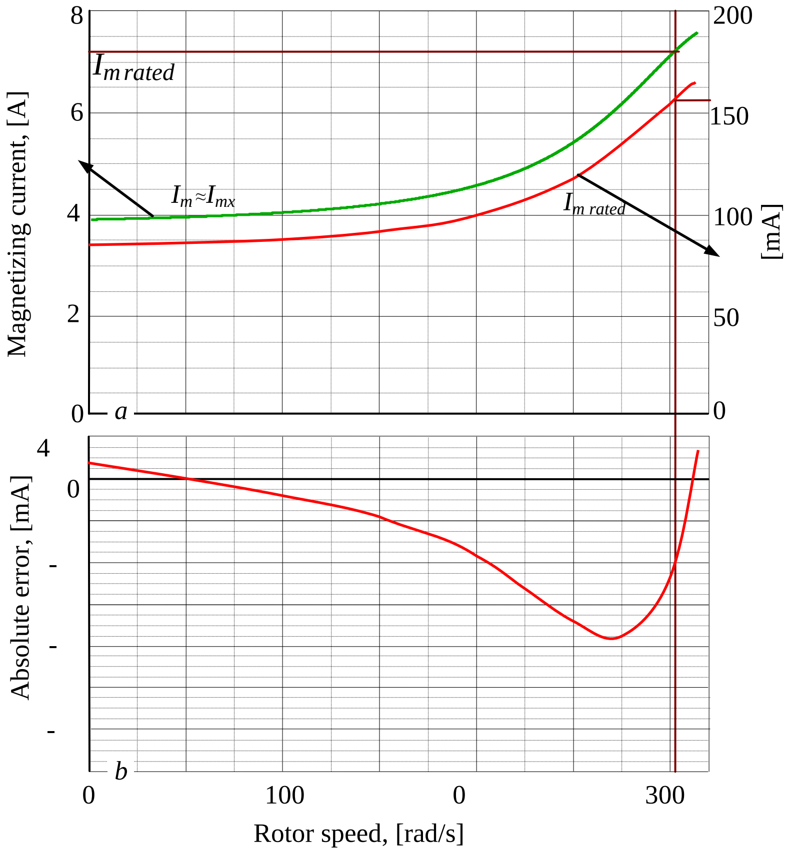

Fig. 5 – Magnetizing current versus speed: *a* – current value; *b* – absolute error.

Fig. 5 shows the values of the magnetizing current obtained by using (13) $I_m \approx I_{mx}$, the current corresponding to the iron losses I_{mr} and the errors compared to the exact values obtained by simulation. The reference current is considered the current flowing through the magnetizing inductance I_{mx}. The errors are due primarily to approximation (3) and secondly to approximation

(6). As in the previous cases, very small errors are obtained at starting (s = 1) and at synchronism (s = 0). Greater errors are also obtained close to the break-down slip s_k; the maximum absolute error is $\Delta I_{m\max} = 0.019A$ and the relative error is $\varepsilon_{m\max} = \Delta I_m / I_{mr} = 0.3\%$. In ordinary load operation the relative error is $\varepsilon_2 = \Delta I_2^{'} / I_{2r}^{'} < 0.2\%$.

4. New Rotor Current Expression

The torque-speed characteristic is the basic characteristic to describe the performance of the induction motor. This can be derived from the equivalent circuit of the three-phase induction motor considering the energy balance (Gheorghiu and Fransua, 1971; Lyshevsky 1999):

$$T = \frac{p}{\omega}\frac{p_{j2}}{s} = \frac{p}{\omega}\frac{3R_2^{'}I_2^{'2}}{s} \tag{17}$$

where p is the number of pole pairs and p_{j2} is the rotor power losses. Using (12) we obtain:

$$T = \frac{U_1^2}{s\omega}\frac{3R_2^{'}}{\left(R_1 + c_1R_2^{'}/s\right)^2 + \omega^2\left(L_{\sigma1} + c_1L_{\sigma2}^{'}\right)^2} \tag{18}$$

This formula is rather useful and convenient to determine the torque-speed relation of induction motors when a full detail of the motor equivalent parameters is available. It is rather an academic formula useful in the study of the influence of the constructive parameters of the induction machine on the torque-speed characteristic.

A common approach to determine the torque curve is to use the full Kloss formula (Boldea and Nasar 2002; Wilamowski and Irwin 2011; Gerling 2015; Pichai 2018). The electromagnetic torque T developed by the induction motor at a given speed (or slip) is expressed as a function of break-down (or maximum) torque T_k and break-down slip s_k.

$$T = \frac{2T_k\left(1 + \lambda s_k\right)}{\frac{s}{s_k} + \frac{s_k}{s} + 2\lambda s_k} \tag{19}$$

where:

$$T_k = \frac{3pU_1^2}{2c_1\omega\left[R_1 + \sqrt{R_1^2 + \left(X_{\sigma1} + c_1X_{\sigma2}^{'}\right)^2}\right]} \tag{20}$$

$$s_k = \frac{c_1 R_2^{'}}{\sqrt{R_1^2 + \left(X_{\sigma 1} + c_1 X_{\sigma 2}^{'}\right)^2}} \tag{21}$$

$$\lambda = \frac{R_1}{c_1 R_2^{'}} \tag{22}$$

From (19) and (17) we obtain another mathematical expression of rotor current, different from the classic one:

$$I_2^{'2} = \frac{2\omega}{3p} \frac{T_k\left(1+\lambda s_k\right)}{R_2^{'}} \frac{s}{\frac{s}{s_k} + \frac{s_k}{s} + 2\lambda s_k} \tag{23}$$

Hence the starting current (s =1) results as:

$$I_{2st}^{'2} = \frac{2\omega}{3p} \frac{T_k\left(1+\lambda s_k\right)}{R_2^{'}} \frac{1}{\frac{1}{s_k} + s_k + 2\lambda s_k} \tag{24}$$

and the rotor current relative to the starting current is:

$$I_2^{'2} = I_{2s}^{'2}\left(\frac{1}{s_k} + s_k + 2\lambda s_k\right) \frac{s}{\frac{s}{s_k} + \frac{s_k}{s} + 2\lambda s_k} \tag{25}$$

Although the starting rotor current is not one of the rated values of the induction motor, the catalog provides the starting stator current from which the starting rotor current can be determined. Therefore from Fig. 2 we get the relationship between the stator and rotor currents. For s = 1 we get:

$$I_{2st}^{'2} = I_{1st}^2 \frac{\left|\underline{Z}_m\right|^2}{\left|\underline{Z}_m + \underline{Z}_2^{'}\right|^2} = I_{1st}^2 \frac{X_m^2}{R_2^{'2} + (X_m + X_{\sigma 2}^{'})^2} \tag{26}$$

$$I_2^{'2} = I_{1st}^2 \frac{X_m^2}{R_2^{'2} + (X_m + X_{\sigma 2}^{'})^2} \left(\frac{1}{s_k} + s_k + 2\lambda s_k\right) \frac{s}{\frac{s}{s_k} + \frac{s_k}{s} + 2\lambda s_k} \tag{27}$$

Using the notation:

$$A = \frac{X_m \sqrt{s_k^2 + 2\lambda s_k^2 + 1}}{\sqrt{R_2^{'2} + (X_m + X_{\sigma 2}^{'})^2}} \approx \frac{\sqrt{1+3s_k^2}}{c_2} \tag{28}$$

(27) becomes:

$$I_2^{'} = I_{1st} A \frac{s}{\sqrt{s^2 + 2\lambda s_k^2 s + s_k^2}} \tag{29}$$

The same result could be obtained directly from (12) in a manner similar to that used when processing the torque. But the mathematical expression of the electromagnetic torque of Kloss aspect is classical and can be found in all textbooks. For this reason we preferred to derive the expression of the rotor current starting from Kloss expression.

The mathematical expression (29) has been validated by simulation. Therefore a quasi-stationary operating is considered. The rotor speed increases slowly at constant slope. Within a 9 s time range, the rotor speed increases from 0 to ω_1=314 rad/s.

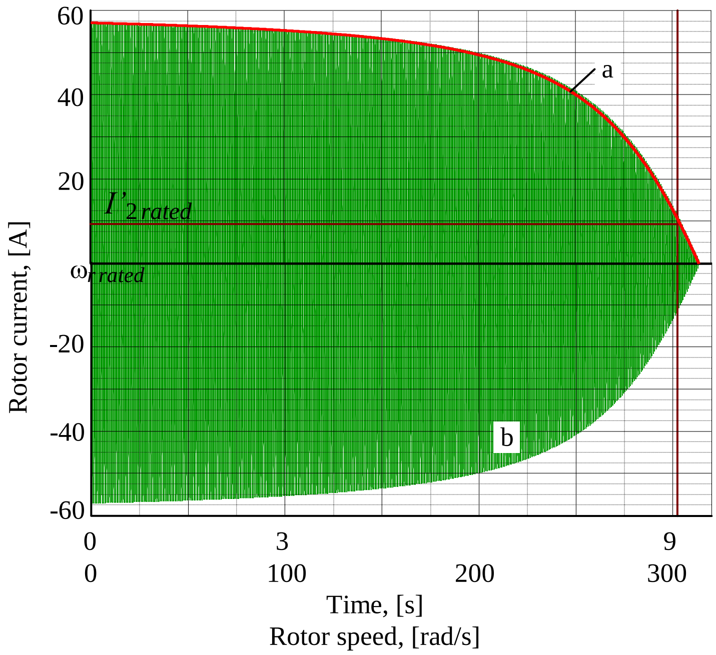

Fig. 6 – a - Rotor current amplitude versus speed calculated with (29);
b - Rotor current in quasi-steady state operation.

The results presented in Fig. 6 highlight the concordance between the values obtained by simulation and those calculated using (29).

5. Conclusions

The stator and rotor currents are usually used to describe the induction machine behavior, so their expressions are well known in the literature. The magnetizing current (13) is less used, therefore its expression has been determined in this paper.

In all current expressions two approximations have been used. The first approximation is almost always involved, so it's not necessarily to recall it. It is about neglecting the iron losses presented in the equivalent electrical scheme by means of the resistance R_m. Mathematically this approximation is presented in (3). The second approximation presented in (6) is unanimously accepted, overlooking the fact that its use aims to simplify the final form of mathematical current expressions.

Although these approximations are accepted, the form of current expressions remains (very) complicated, thus masking the fact that the expressions are not exact.

Based on a real application the paper determines the error of classical expressions of currents (11), (12) and (13). In all cases in starting operation (s = 1) and at synchronism (s = 0) the errors are very small $\varepsilon < 0.1\%$ and therefore negligible. The largest errors are obtained close to the break-down slip s_k and can reach the value $\varepsilon \approx 2\%$ in the case of stator current, smaller for the rotor current, $\varepsilon \approx 1\%$ and only $\varepsilon \approx 0.3\%$ for the magnetizing current. Under normal load operating, $s_r < s < 0$, the errors do not exceed 1% and can be considered negligible for almost all applications.

Starting from Kloss's expression of the electromagnetic torque, the paper derives a new mathematical expression of the rotor current (29). This does not differ in essence from (12) but better highlights the dependence of the rotor current on the slip. The expression is suitable to practical applications because it uses known or determinable parameters of the asynchronous machine, such as s_k, λ, I_{1st}, A.

REFERENCES

Agamloh E., Cavagnino A., Vaschetto S., *Accurate Determination of Induction Machine Torque and Current versus Speed Characteristics*, IEEE Energy Conversion Congress and Exposition, 1-8 (2016).

Aree P., *Analytical Determination of Speed-Torque and Speed-Current Curves of Single-Cage Induction Motor under Supply Voltage and Frequency Variations*, COMPEL – The International Journal for Computation and Mathematics in Electrical and Electronic Engineering (2018).

Boldea I., Nasar S.A., *The Induction Machine Handbook*, CRC Press, New York, NY, 164 (2002).

Cociu L., Cociu R., *Aspects Regarding Pspice Electrical Machines Simulation.* The 5th International Conference on Electromechanical and Power Systems SIELMEN'05, Chişinău, Rep. Moldova, 6-8 oct. 2005, Vol. **II**, 842-845, ISBN 973-716-230-7.

Cociu V.R., Cociu L., *An Optimized Pspice Implementation of the Three+Phase Induction Machine Model*, Bul. Inst. Polit. Iaşi, **LVII(LXI)**, *5*, s. Electrotehnică, Energetică, Electronică, 103-112 (2011).

Cociu V.R., Cociu L., *Symmetrical Components Method in Induction Machine Behavior Analisis*, Bul. Inst. Polit. Iaşi, **LX(LXIV)**, *4*, s. Electrotehnică, Energetică, Electronică, 23-38 (2014).

Gerling D., Electrical Machine, Springer-Verlag, Berlin, 477-480 (2015).

Gheorghiu I.S., Fransua A.S., *Tratat de maşini electrice, Maşini asincrone*, Ed. Academiei RSR, Bucureşti, 1971.

Justus O., *Dynamisches Verhalten elektrischer Maschinen. Eine Einfuhrung in die numerische Modellierung mit PSPICE*, Vieweg & Sohn Verlagsges. Braunschweig/Wiesbaden, 1993.

Lyshevsky S.E., *Electromechanical Systems, Electric Machines and Applied Mechatronics*, CRC Press, Boca Raton, Florida, 1999.

Popa V.M., *Receptoare generalizate in electrotehnica*, Ed. Univ. "Lucian Blaga", Sibiu, 2010.

Rockwell Automation, *Application Basic of Operation of Three-Phase Induction Motors: Design Duty Types Selection Dimensioning*, Sprecher+Schuh AG Rockwell Automation, Aarau (1996).

Wilamowski B.M., Irwin J.D., *The Industrial Electronic Handbook: Power Electronics and Motor Drives*, CRC Press, FL, pp. 154-157, (2011).

CONSIDERAŢII ASUPRA EXPRESIEI CURENŢILOR FUNCŢIE DE VITEZA UNGHIULARĂ LA MAŞINA ASINCRONĂ

(Rezumat)

Lucrarea prezintă o analiză a expresiilor clasice ale curenţilor prin maşina asincronă. Se evidenţiază simplificările utilizate şi se evaluează erorile datorate acestor simplificări. Pentru un exemplu concret, se calculează erorile absolute având ca valoare de comparaţie rezultatele obţinute prin simulare pe baza modelului complet bifazat sau trifazat. Se fac comentarii asupra valorilor erorilor absolute şi relative. În final, pornind de la expresia clasică (simplificată) a cuplului electromagnetic se propune o nouă expresie a curentului rotoric care pune în evidenţă în primul rând modul de variaţie al curentului funcţie de viteza rotorică (alunecare).