BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iaşi
Tomul LVII (LXI), Fasc. 5, 2011
Secţia
ELECTROTEHNICĂ. ENERGETICĂ. ELECTRONICĂ

EQUALIZATION OF A CHAOTIC DISCRETE-TIME COMMUNICATION SYSTEM

BY

VICTOR GRIGORAŞ[1] and CARMEN GRIGORAŞ[2,*]

[1]“Gheorghe Asachi” Technical University of Iaşi
Faculty of Electronics, Telecommunications and Information Technology,
[2]“Gr.T. Popa” Medicine and Pharmacy University of Iaşi
and
Romanian Academy, Iaşi Branch,
Institute of Computer Science

Received: November 3, 2011
Accepted for publication: November 27, 2011

Abstract. This paper presents the possibility of designing a linear, discrete-time communication system based on nonlinear modulation of the modulating signal onto a chaotic emitter aiming at wide-band secure transmission of information. After presenting the modified Chen system setup, conditions for correct demodulation and linear dynamic, input–output behavior are studied. For a linear dynamic relation between the modulating and demodulated signals, channel equalization is used to achieve wider bandwidth transmission. The presented simulation results highlight the applicability of the proposed method for high speed digital communication. The overall performance of the resulting communication system is analysed in terms of speed, security and occupied frequency bandwidth.

Key words: chaos synchronization; feed forward equalization; Chen system.

1. Introduction

The research field of secure communication based on chaos synchronization and modulation has received much attention in the last years.

---

[*] Corresponding author: *e-mail*: crmngrig@gmail.com

Transmitting the information signal on a chaotic carrier, generated by a nonlinear emitter system and demodulating it with a synchronizing receiver is a sensitive process, especially in the case of analog communications, affected by several non-idealities that disturb exact recovery of the transmitted data. Among these non-idealities most noticeable are parameter mismatch between emitter and receiver and channel noise and perturbation that both lead to noisy data recovery. Another type of error in data transmission based on nonlinear modulation is distortion, both nonlinear and amplitude and phase types. Linear distortion on analog transmission systems based on nonlinear modulation was reported in previous research (Grigoraş, 2009). Due to the generalization of digital communication and better nonlinear system resistance to noise and parameter mismatch, the present contribution focuses on a discrete-time approach aiming at characterizing a chaotic modulation method to build a wide-band communication channel. The general synchronization setup is based on the system partitioning method, first introduced by Carroll and Pecorra (1991). The discrete-time nonlinear emitter is designed by digitizing an analog, modified Chen, prototype. The relationship between the modulating signal and the demodulated one is linear and dynamic. The low-pass characteristic of the abovementioned relationship shows the frequency limitations for the modulating signal. In order to increase the transmitted signal bandwidth, to achieve the desired performance in practical applications, a feed forward equalizer is proposed and its efficiency studied. The modified Chen system is chosen for implementation advantages and for synchronization simplicity.

The next section concentrates on the demonstration of the proposed method. The error dynamics method is used, leading to the conclusion that the input–output relation of the proposed channel is linear. Simulation results, confirming the theoretical solution, are presented in the third section. Some short concluding remarks end our contributions.

2. The Principle of the Method

The discrete-time, secure communication system proposed in this paper is built starting from the modified Chen (2000) analogue prototype

$$\begin{cases} x' = ay - ax, \\ y' = dx - cy - xz,, \\ z' = -bz + xy. \end{cases} \tag{1}$$

The authors have demonstrated (2009) that, for a large enough coefficient range around the nominal values (a = 15, b = 3, c = 1 and d = 32), the analogue modified Chen system exhibits chaotic behavior. Starting from this analogue prototype and applying the forward Euler numeric integration method we can develop a discrete-time chaotic system. For a small enough value of the discrete-time step, T_s, compared to the analogue time constants of the analogue prototype, the resulting discrete-time system is described by the state equations

$$\begin{cases} x[k+1] = x[k] + T_s\left(ay[k] - ax[k]\right), \\ y[k+1] = y[k] + T_s\left(dx[k] - cy[k] - x[k]z[k]\right), \\ z[k+1] = z[k] + T_s\left(x[k]y[k] - bz[k]\right). \end{cases} \quad (2)$$

In order to design a synchronizing receiver for our transmission purposes, we chose to transmit the y state variable, and apply the system partitioning method leading to the state equations

$$\begin{cases} \tilde{x}[k+1] = \tilde{x}[k] + T_s\left(ay[k] - a\tilde{x}[k]\right), \\ \tilde{z}[k+1] = \tilde{z}[k] + T_s\left(\tilde{x}[k]y[k] - b\tilde{z}[k]\right), \end{cases} \quad (3)$$

where the signals and functions marked with tilde ($\tilde{.}$) denote the receiver correspondents of the emitter ones.

To demonstrate synchronization for this type of receiver system we take into account the autonomous case, $y[k] = 0$. The resulting state equations are linear, with a state transition matrix of the form

$$\mathbf{A} = \begin{pmatrix} 1 - T_s a & 0 \\ 1 & 1 - T_s b \end{pmatrix}. \quad (4)$$

By choosing a small enough value of the discrete-time step, T_s, we can force the system eigenvalues

$$\mathbf{\Lambda} = \begin{pmatrix} 1 - T_s a \\ 1 - T_s b \end{pmatrix}, \quad (5)$$

well within the unit circle of the complex plane, ensuring global exponential asymptotic stability of the receiver.

The next step is to develop the error dynamics and check it for stability. The error vector is the difference between the emitter state vector and the receiver counterpart

$$\boldsymbol{\varepsilon}[k] = \mathbf{x}[k] - \tilde{\mathbf{x}}[k]. \quad (6)$$

Due to the error correction algorithms encountered in digital transmission we can consider discrete-time channel noise and distortion as null, and take into account the parameter matching hypothesis because of the high precision of digital implementation. Subtracting the state equations of the receiver from the first and third counterparts at the emitter we obtain the error state equations

$$\begin{cases} \varepsilon_1^{'} = -a\varepsilon_1, \\ \varepsilon_3^{'} = -b\varepsilon_3 + \varepsilon_1 y. \end{cases} \tag{7}$$

The state errors, $\varepsilon_1 = x - \tilde{x}$, $\varepsilon_3 = z - \tilde{z}$, exponentially converge to zero, with the time constants $-1/a$ and $-1/b$.

The modulation part of the system design is performed by adding the modulating signal in the equation of the transmitted signal

$$\begin{cases} x[k+1] = x[k] + T_s(ay[k] - ax[k]), \\ y[k+1] = y[k] + T_s(dx[k] - cy[k] - x[k]z[k]) + m[k], \\ z[k+1] = z[k] + T_s(x[k]y[k] - bz[k]). \end{cases} \tag{8}$$

At the receiving end of the communication channel we demodulate the received signal, $y[k]$, by subtracting from it the locally recovered second state variable, $\tilde{y}[k]$,

$$\tilde{y}[k+1] = \tilde{y}[k] + T_s(d\tilde{x}[k] - c\tilde{y}[k] - \tilde{x}[k]\tilde{z}[k]). \tag{9}$$

The error dynamics for the second state variable leads to the difference eq. for the demodulated signal

$$\tilde{m}[k+1] = (1 - cT_s)\tilde{m}[k] - T_s((z[k] - d)\varepsilon_1[k] - \tilde{x}[k]\varepsilon_3[k]) + m[k]. \tag{10}$$

Due to the exponentially global asymptotic stability of the error dynamics, after a short transient, with the time constant τ_{Tr} = Max($-1/a$, $-1/b$), the errors $\varepsilon_1 = x - \tilde{x}$, $\varepsilon_3 = z - \tilde{z}$ decay to zero and the demodulator equation can be approximated with

$$\tilde{m}[k+1] = (1 - cT_s)\tilde{m}[k] + m[k]. \tag{11}$$

Due to the linear memory character of the model, it can be characterized, in the z transform domain, by the transfer function

$$H(z) = \frac{\tilde{M}(z)}{M(z)} = \frac{1}{z - (1 - cT_s)}. \tag{12}$$

The transfer function (12) is a low-pass, first order one, limiting the recoverable frequency bandwidth of the modulating signal. To increase the bandwidth of the transmitted signal we can use a feed-forward equalizer with a transfer function of the form

$$H_E(z) = k\frac{z-(1-cT_s)}{z-(1-kcT_s)} = 100\cdot\frac{z-(1-cT_s)}{z-(1-100cT_s)}. \tag{13}$$

The real zero of the equalizer transfer function compensates the pole of the communication system linear model, while the new pole limits the overall bandwidth to avoid excessive high frequency synchronization error to affect the recovered signal. Thus, the overall transfer function is also a low-pass one, but with a higher cut-off frequency

$$H_{\text{tot}}(z) = H(z)H_E(z) = \frac{1}{z-(1-kcT_s)} = \frac{1}{z-(1-100cT_s)}. \tag{14}$$

3. Simulation Results

In order to achieve secure communication we have to verify the chaotic behavior of the proposed discrete-time version of the modified Chen system. In Fig. 1 it is represented the strange attractor resulting for the nominal values of the parameters confirming the chaotic hypothesis.

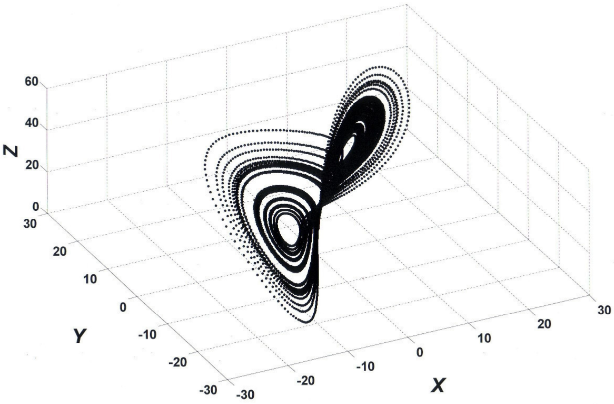
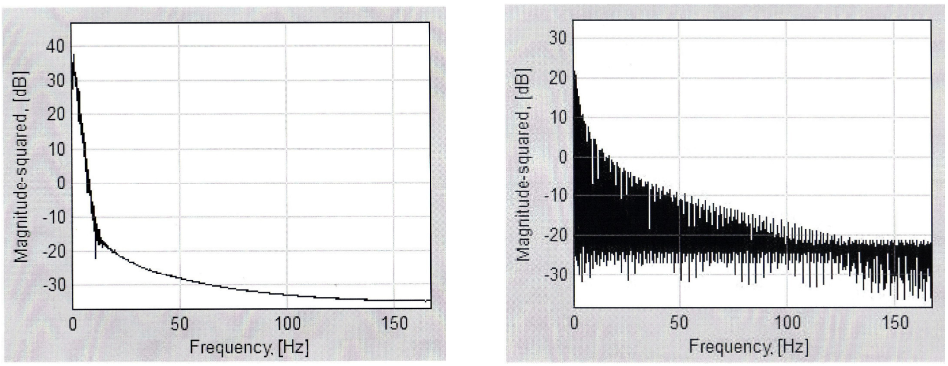


Fig. 1 – The discrete-time modified Chen strange attractor.

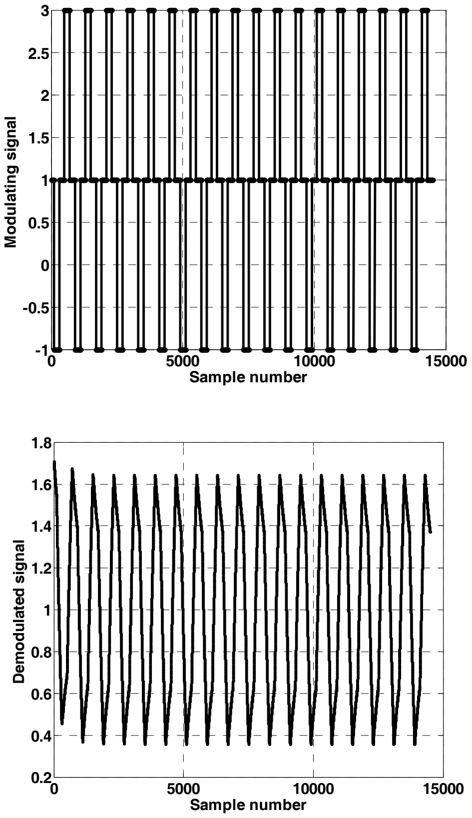
In order to track the visibility of the modulating signal in the transmitted chaotic one, we studied the power spectral density (PSD) of the transmitted signal and the modulating signal. Simulation results are depicted in Fig. 2, showing the independence of the chaotic spectrum of the modulating signal one.

In Fig. 3 we give an example of modulating signal and recovered one. The demodulated signal waveform highlights the linear, low-pass characteristic of the proposed transmission system. Implementing the proposed feed forward equalizer, we obtain the results depicted in Fig. 4. The demodulated signal at the equalizer output is far closer to the modulating signal.



a *b*

Fig. 2 – Spectrum analysis: transmitted signal (*a*); modulating signal (*b*).



a

b

Fig. 3 – The modulating signal (*a*); the demodulated signal without equalizer (*b*).

Still, the overall transfer function has a low-pass selectivity, albeit with a higher cut-off frequency, so the demodulation error is not null. The very short peaks present in the Fig. 4 *b* are the result of front smoothing due to the resulting selective effect.

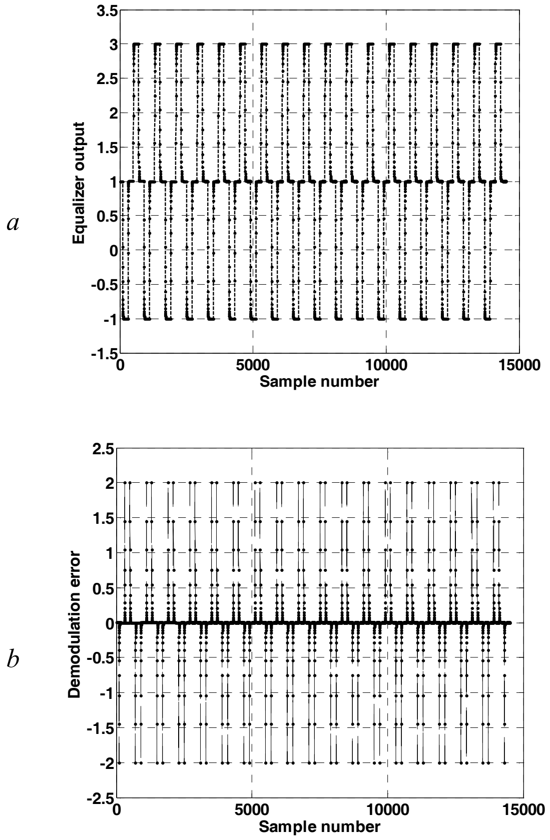


Fig. 4 – The demodulated signal with equalizer (*a*); the demodulation error (*b*).

5. Conclusions

We proposed a discrete-time wide-band communication system based on nonlinear synchronization and direct modulation. Using the system partitioning synchronization and error dynamics methods, we demonstrated that a linear dynamic relation exists between the modulating signal and its demodulated counterpart for the discrete version of the modified Chen system. A feed-forward channel equalization technique ensures that the modulating signals can have large enough bandwidth, leading to the possibility of high speed digital communication. The presented simulation results highlight the feasibility of the general method.

REFERENCES

Carrol T.L., Pecorra L.M., *Synchronizing Chaotic Circuits*. IEEE Trans. on Circ. a. Syst., **38**, *4*, 453-456 (1991).

Grigoraş V., Grigoraş C., *Linear Communication Channel Based on Chaos Synchronization*. Proc. of the 2nd Chaotic Model. a. Simul. Internat. Conf., Chaos2009, June, 1-4, 2009, Chania, Crete, Greece, 24-30.

Grigoraş V., Tătaru V., Grigoraş C., *Chaos Modulation Communication Channel: A Case Study*. Proc. of the Internat. Symp. on Signals Circ. a. Syst., ISSCS2009, July, 9-10, 2009, Iaşi, Romania, 489-492.

Ueta T., Chen G., *Bifurcation Analysis of Chen's Equation*. Internat. J. of Bifurc. a. Chaos, **10**, 1917-1931 (2000).

EGALIZAREA UNUI SISTEM DE COMUNICAŢII HAOTIC DISCRET

(Rezumat)

Se prezintă posibilitatea proiectării unui sistem de comunicaţii discret, liniar, bazat pe modulaţia neliniară a semnalului modulator pe un emiţător haotic vizând transmiterea securizată a informaţiei în bandă largă. După prezentarea structurii sistemului Chen modificat sunt demostrate condiţiile pentru demodularea corectă şi comportarea dinamică liniară intrare–ieşire a acestuia. În condiţiile legăturii dinamice liniare între semnalul modulator şi cel demodulat, se utilizează un egalizor in cascadă pentru realizarea unei transmisii de banda lărgită. Rezultatele de simulare prezentate subliniază aplicabilitatea metodei propuse la transmisia digitală de mare viteză. Performanţele globale ale sistemului de comunicaţii rezultat sunt analizate prin prisma vitezei, securităţii şi a benzii de frecvenţe ocupată.