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DISCRETE-TIME HYPERCHAOTIC SYSTEM SYNCHRONIZATION

BY

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Abstract. Nonlinear systems with hyperchaotic dynamical behaviour can be used in signal encryption. The present contribution presents the design of hyperchaotic discrete-time emitter circuit the synchronizing receiver and their dynamics. Possible application in wide-band signal transmission is presented by using error dynamics method.

Keywords: hyperchaotic systems; chaotic dynamics; discrete-time systems; signal encryption.

1. Introduction

Nonlinear systems perform complex dynamics including chaotic and hyperchaotic ones. Such systems were implemented using analog (Sambas *et al*., 2015) or digital (Bouraoui *et al*., 2013) circuits. Previously published results

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highlight the fact that complex emitter dynamics provide high security of the chaotically encrypted signal (Grigoraş and Grigoraş, 2017; Maqbool *et al*., 2017) leading to the study of hyperchaotic discrete-time systems as emitting element in the communication structure. Using the proposed approach to biomedical signal transmission, is confirmed in (Beck *et al*., 2021; Liao *et al*., 2021).

The present paper presents the design of a secure transmission system based on a discrete-time hyperchaotic emitter and a synchronizing receiver. The performance of the proposed method is analyzed dynamically and statistically for the digital implementation suggested in (Grigoraş and Grigoraş, 2018). The use of the proposed communication pair to wide-band signal transmission is also pressented.

The following section presents the design of the nonlinear emitter. Section three aims at the design of the discrete synchronizing receiver. Digital implementation results, confirming the desired dynamic and synchronizing behavior are presented in the following section. The final section highlights the resulting conclusions of the presented research.

2. Discrete Emitter System

The proposed discrete-time nonlinear emitter is designed based on the analogue hyperchaotic prototype system, developed in (Ma and Zang, 2013):

$$\begin{cases} dx/dt = a\cdot x - a\cdot y + w - y\cdot z \\ dy/dt = -b\cdot y + x\cdot z \\ dz/dt = d\cdot x - c\cdot z + x\cdot y \\ dw/dt = -e\cdot x - e\cdot y \end{cases} \tag{1}$$

The analogue emitter, useful in secure communication systems, is modified by adding a supplementary term to the fourth state equation:

$$\begin{aligned} &(E_a:)\begin{cases} dx_E/dt = a\cdot x_E - a\cdot y_E + w_E - y_E\cdot z_E \\ dy_E/dt = -b\cdot y_E + x_E\cdot z_E \\ dz_E/dt = d\cdot x_E - c\cdot z_E + x_E\cdot y_E \\ dw_E/dt = -e\cdot x_E - e\cdot y_E - f\cdot w_E \end{cases} \\ &Out(t) = x_E(t) \end{aligned} \tag{2}$$

In order to obtain a discrete-time nonlinear hyperchaotic system, with the same order as the analogue prototype, we choose to use the Euler approximation method:

$$\frac{dx}{dt} \approx \frac{1}{T}\left(x(t_{k+1}) - x(t_k)\right) \tag{3}$$

In Eq. (3), x is a generic state variable and T is the timing period of the discrete-time emitter system, equal to the sampling period of the discrete-time processed signals:

$$x[k] = x(t_k); \quad x[k+1] = x(t_{k+1}); \quad T = t_{k+1} - t_k \quad k \in Z. \tag{4}$$

By applying the approximations (3) and (4) to the analogue emitter state Eqs. (2), we obtain the state equations of the discrete-time emitter, (E_d:):

$$\left(E_d :\right)\begin{cases} x[k+1] = x[k] + T\cdot\left(a\cdot x[k] - a\cdot y[k] + w[k] - y[k]\cdot z[k]\right) \\ \mathrm{y}[k+1] = y[k] + T\cdot\left(-b\cdot y[k] + x[k]\cdot z[k]\right) \\ \mathrm{z}[k+1] = z[k] + T\cdot\left(d\cdot x[k] - c\cdot z[k] + x[k]\cdot y[k]\right) \\ \mathrm{w}[k+1] = w[k] + T\cdot\left(-e\cdot x[k] - e\cdot y[k] - f\cdot w[k]\right) \end{cases} \tag{5}$$

$$Out[k] = x[k]$$

Starting from the parameters of analogue state equations in (Ma and Yang, 2013):

$$a = 0.89\; b = 9\; c = 50\; d = 0.06\; e = 0.9 \tag{6}$$

we make a parametric analysis for the discrete emitter, obtaining the bifurcation diagrams similar to the examples depicted in Fig. 1 to Fig. 6:

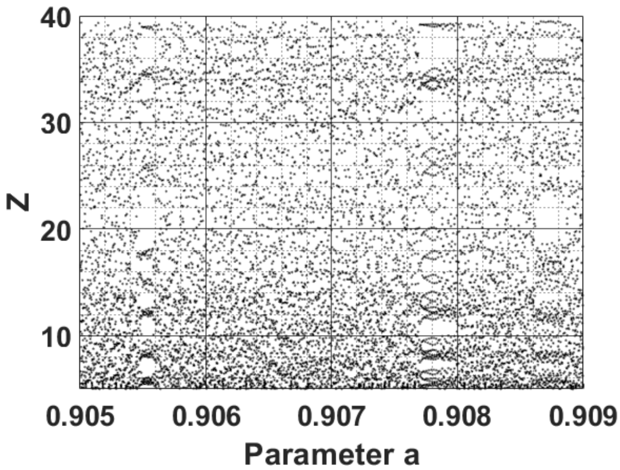

Fig. 1 – Bifurcation diagram of the state variable z for the parameter a.

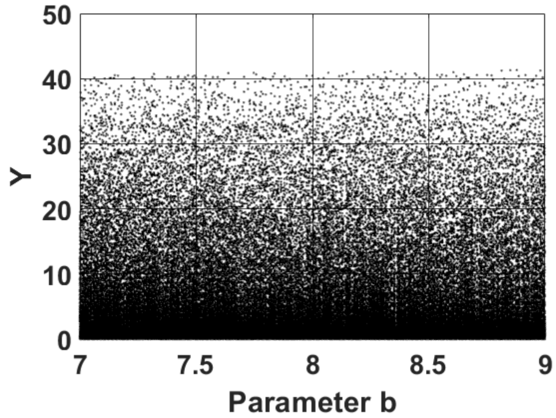

Fig. 2 – Bifurcation diagram of the state variable *y* for the parameter *b*.

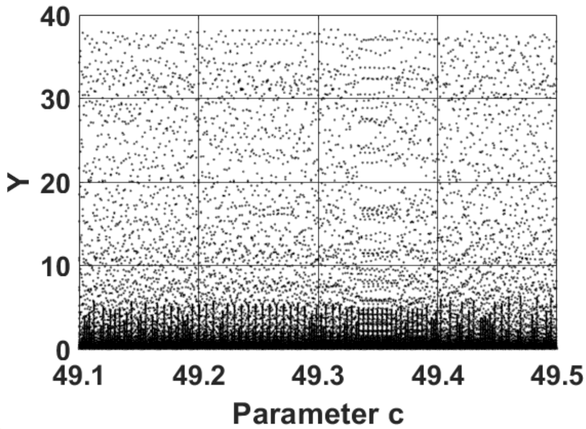

Fig. 3 – Bifurcation diagram of the state variable *y* for the parameter *c*.

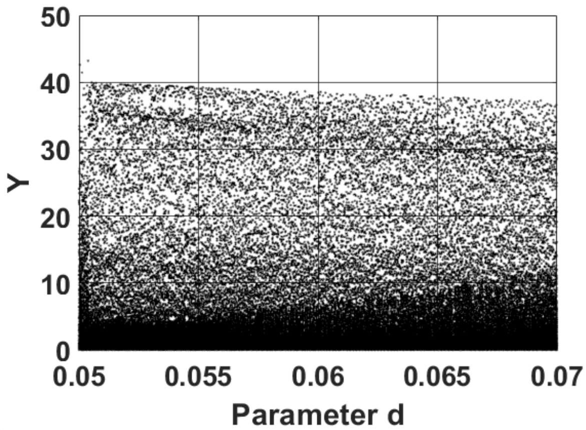

Fig. 4 – Bifurcation diagram of the state variable *y* for the parameter *d*.

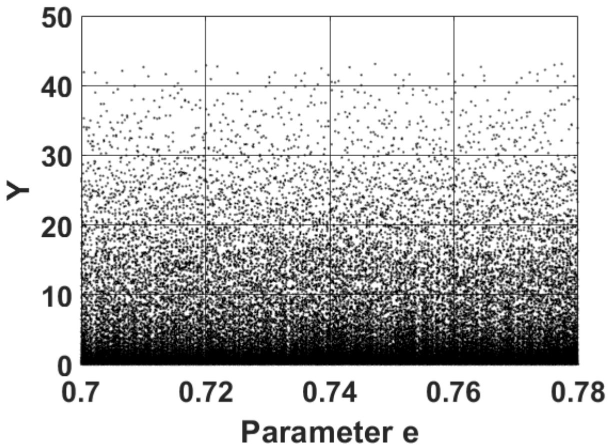

Fig. 5 – Bifurcation diagram of the state variable *y* for the parameter *e*.

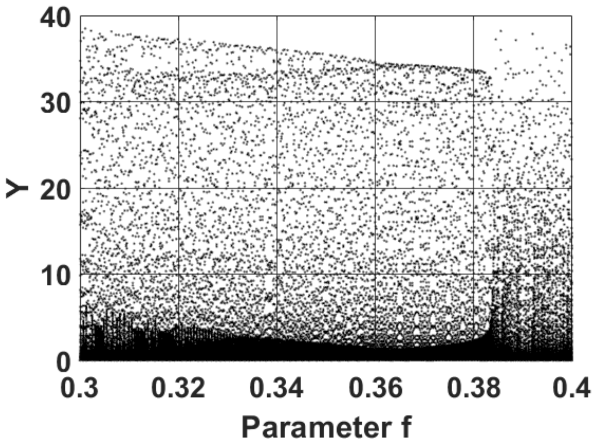

Fig. 6 – Bifurcation diagram of the state variable *y* for the parameter *f*.

The bifurcation diagrams examples show the wide range of parameter values enabling hyperchaotic dynamic behavior, with small value intervals with periodic behavior, such as 0.9055 and 0.9079 for the ‘*a*’ parameter (Fig. 1) and 49.35 for the ‘*c*’ parameter (Fig. 3).

3. Synchronizing Receiver Design

The synchronizing receiver corresponding to the designed emitter may be developed using the emitter partitioning method, leading to the discrete-time state Eqs. (7):

$$
\begin{aligned}
&Rec[k] = Out[k] = x[k] \\
&(R_d :)\begin{cases} y_R[k+1] = y_R[k] + T\cdot\left(-b\cdot y_R[k] + Rec[k]\cdot z_R[k]\right) \\ z_R[k+1] = z_R[k] + T\cdot\left(-c\cdot z_R[k] + d\cdot Rec[k] + Rec[k]\cdot y_R[k]\right) \\ w_R[k+1] = w_R[k] + T\cdot\left(-e\cdot y_R[k] - f\cdot w_R[k] - e\cdot Rec[k]\right) \end{cases}
\end{aligned} \tag{7}
$$

The linear part of the discrete receiver state equations is presented in Eq. (8). These equations were obtained by eliminating the received signal, *Rec*[*k*], in order to highlight the receiver dynamical behavior.

$$
\begin{aligned}
&Rec[k] = 0 \\
&(R_{Lin} :)\begin{cases} y_R[k+1] = (1 - T\cdot b)\cdot y_R[k] \\ z_R[k+1] = (1 - T\cdot c)\cdot z_R[k] \\ w_R[k+1] = -T\cdot e\cdot y_R[k] + (1 - T\cdot f)\cdot w_R[k] \end{cases}
\end{aligned} \tag{8}
$$

The linearized receiver, (R_{lin}), state Eqs. (8) lead to the state transition matrix, **A**:

$$
\mathbf{A} = \begin{pmatrix} 1 - b\cdot T & 0 & 0 \\ 0 & 1 - c\cdot T & 0 \\ -e\cdot T & 0 & 1 - f\cdot T \end{pmatrix} \tag{9}
$$

The resulting state transition matrix eigenvalues have subunitary modulus for properly chosen coefficient values:

$$
\begin{aligned}
&\begin{pmatrix} \lambda_y & \lambda_z & \lambda_w \end{pmatrix}^T = \begin{pmatrix} 1 - b\cdot T & 1 - c\cdot T & 1 - f\cdot T \end{pmatrix}^T \\
&b, c, f \in \left(0, \ldots, 2/T\right)
\end{aligned} \tag{10}
$$

This ensures the stability of (R_{lin}) system, thus fulfilling a necessary condition for synchronization. To fulfill the sufficient condition, the error dynamics between emitter and receiver can be studied. The error vector does not contain the transmitted signal, resulting in the form:

$$
\boldsymbol{\varepsilon}^T = \left[\varepsilon_y\ \varepsilon_z\ \varepsilon_w\right]^T = \left[y\ z\ w\right]^T - \left[y_R\ z_R\ w_R\right]^T \tag{11}
$$

By subtracting the discrete time state equations of the receiver (7) from their counterparts of the emitter (5), the difference state equations that model the time evolution of the error result in the form (12):

$$(\varepsilon:)\begin{cases}\varepsilon_y[k+1]=(1-T\cdot b)\cdot\varepsilon_y[k]+T\cdot Rec[k]\cdot\varepsilon_z[k]\\ \varepsilon_z[k+1]=(1-T\cdot c)\cdot\varepsilon_z[k]+T\cdot d\cdot Rec[k]+T\cdot Rec[k]\cdot y_R[k]\\ \varepsilon_w[k+1]=(1-T\cdot f)\cdot\varepsilon_w[k]-T\cdot e\cdot\varepsilon_y[k]-T\cdot e\cdot Rec[k]\end{cases}\tag{12}$$

Eliminating the transmitted signal from the error difference state equations, we highlight only the error dynamical evolution, obtaining the linear first order difference equations:

$$(\varepsilon:)\begin{cases}\varepsilon_y[k+1]=(1-T\cdot b)\cdot\varepsilon_y[k]\\ \varepsilon_z[k+1]=(1-T\cdot c)\cdot\varepsilon_z[k]\\ \varepsilon_w[k+1]=(1-T\cdot f)\cdot\varepsilon_w[k]-T\cdot e\cdot\varepsilon_y[k]\end{cases}\tag{13}$$

This is highly similar to the dynamical behavior of the linearized receiver (8), leading to the conclusion of the error stability, shown by the graphical example in Fig. 7:

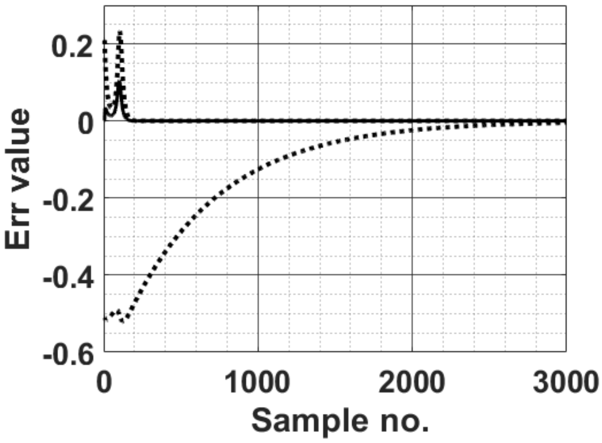

Fig. 7 – State synchronization errors for *y*[k], *z*[k] and *w*[k].

4. Digital Implementation Results

Aiming to use the synchronizing pair (*Ed*) – (*Rd*), (5) and (7), for secure transmission of discrete time modulating signal, the direct modulation method is used. The modulating signal, *m*[*k*], is added to the first equation of the emitter, leading to a dynamically modulated, in a nonlinear way, transmitted signal *Out*[*k*] = *x*[*k*] = *Rec*[*k*]. The resulting modulated discrete-time emitter state equations are (14):

$$\left(E_{\text{mod}}:\right)\begin{cases} x[k+1]=\left(1+T\cdot a\right)\cdot x[k]-T\cdot a\cdot y[k]+T\cdot w[k] \\ \qquad -T\cdot y[k]\cdot z[k]+m[k] \\ \mathrm{y}[k+1]=\left(1-T\cdot b\right)\cdot y[k]+T\cdot x[k]\cdot z[k] \\ \mathrm{z}[k+1]=\left(1-T\cdot c\right)\cdot z[k]+T\cdot d\cdot x[k]+T\cdot x[k]\cdot y[k] \\ \mathrm{w}[k+1]=\left(1-T\cdot f\right)\cdot w[k]-T\cdot e\cdot x[k]-T\cdot e\cdot y[k] \end{cases} \tag{14}$$
$$Out[k]=x[k]$$

The digital implementation of the discrete-time hyperchaotic emitter, is based on a 32-bit processor having an ARM Cortex M7 architecture. This processor performs the required calculations fast enough for 40 kHz signal processing. The 12 bits D/A converters ensure also the necessary timing in the interfacing process.

The resulting pseudo code is:

```
declare A/D input, D/A output
initialize (actual emitter vector);
while switch not pressed:
      input A/D mod signal sample;
      compute state function
      (actual emitter vector, mod signal sample);
      store result in (next emitter vector);
      transfer
      (next emitter vector) > (actual emitter vector);
      output D/A transmit signal sample;
end while;
```

The data structures needed for this code are based upon two double vectors with four components:

```
      actual emitter vector;
      next emitter vector;
```

Added to these vectors are the double variables:

T, a, b, c, d, e, f

The receiver (7) can demodulate the received signal, *Rec*[*k*] = *Out*[*k*], by subtracting a recovered first state variable, x_R[k], at the receiver end from the corresponding emitter one, *x*[k].

$$\begin{aligned}&\left(Demod:\right) x_R[k+1] = \left(1+T\cdot a\right)\cdot x_R[k] - T\cdot a\cdot y_R[k] + \\ &\qquad T\cdot w_R[k] - T\cdot y_R[k]\cdot z_R[k] \\ &\tilde{m}[k] = x[k] - x_R[k]\end{aligned} \tag{15}$$

In a similar way that the modulated emitter (14) was implemented by program on the digital processor, the discrete synchronizing receiver (7) with the demodulator (15) was programmed on the used microcontroller system with the pseudo code:

```
declare A/D input, D/A output
initialize (actual receiver vector);
while switch not pressed:
      input A/D transmit signal sample;
      compute state function
      (actual receiver vector,transmit signal sample);
      store result in (next receiver vector);
      compute demod signal sample;
      transfer
      (next receiver vector)> (actual receiver vector)
      output D/A demod signal sample;
end while;
```

The data structures implemented in this algorithm are based upon two double four dimension vectors:

```
      actual receiver vector;
      next receiver vector;
```

and the scalar variables system parameters:

T, a, b, c, d, e, f ;

The demodulated signal, $\tilde{m}[k]$, approaches the values of the modulating signal, $m[k]$, after the synchronization transient has faded, as suggested in the example in Fig. 8, where a sine modulating signal was used.

For wider band signals, the graphical representations in Fig. 9 show an example of transmitting periodic square shaped signal, highlighting the fact that the larger modulating signal spectrum leads to somewhat larger demodulating error, especially at the faster variation of the rectangular fronts.

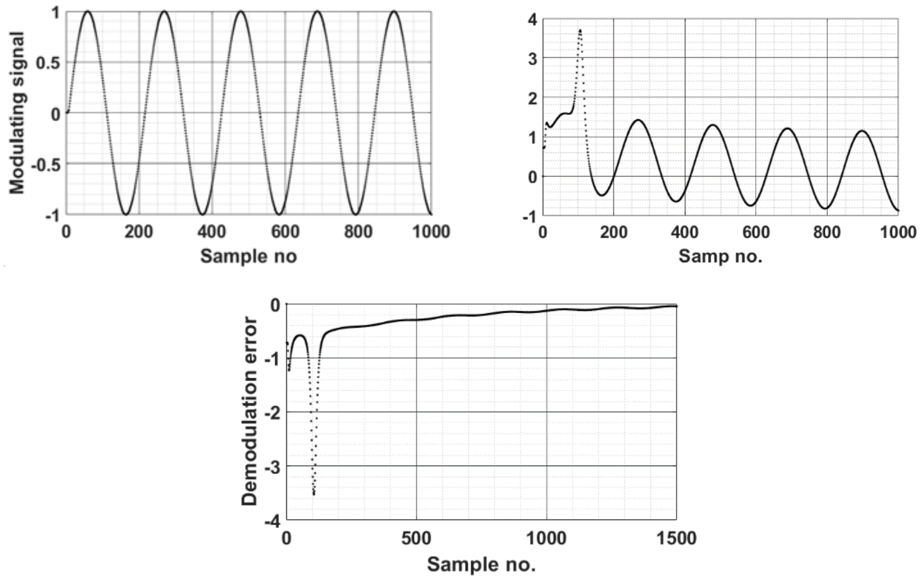

Fig. 8 – Modulation signal, demodulated signal
and demodulation error.

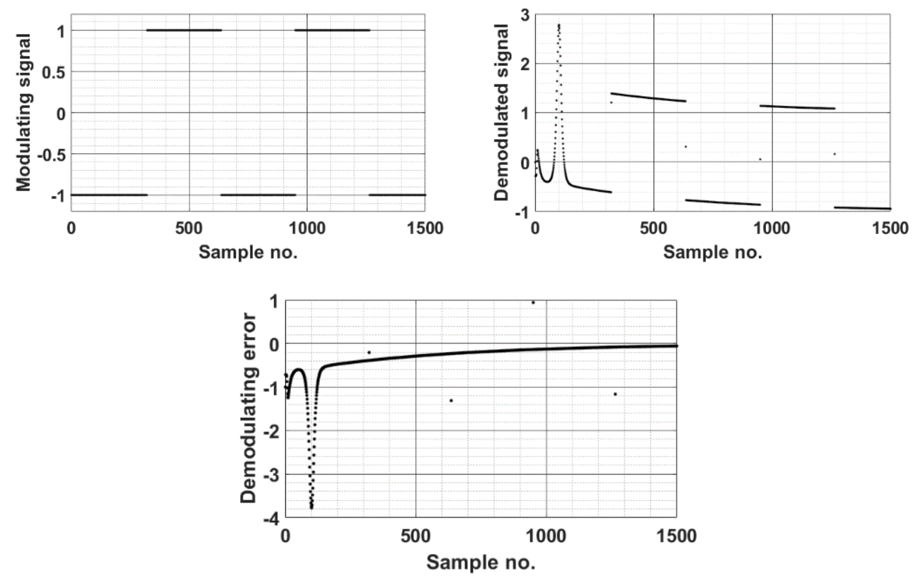

Fig. 9 – Modulating square signal, demodulated signal
and demodulation error.

5. Conclusion

The present contribution shows the design of a discrete-time communication channel developed with a nonlinear modulation of a hyperchaotic emitter and an emitter partitioning type synchronizing receiver. The emitter development starts from a previously published hyperchaotic analogue system, with slight modifications, aiming a more direct design of the synchronizing receiver. The dynamical behavior of the modified emitter is detailed using computer simulations and confirmed by the programable digital implementation. The synchronizing receiver is shown to be stable and both synchronization and external signal modulation are shown to function correctly on the microcontroller processor.

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SINCRONIZAREA SISTEMELOR HIPERHAOTICE ÎN TIMP DISCRET

(Rezumat)

Sistemele neliniare cu o comportare dinamică hiperhaotică sunt studiate pentru aplicaţii în măsurări complexe, modulaţie şi criptare. Semnalele unidimensionale pot fi transmise securizat folosind criptarea haotică a sistemelor neliniare discrete. Lucrarea prezentă analizează sincronizarea hiperhaotică aplicată în comunicaţii securizate. Performanţa perechii emiţător-receptor este prezentată în cazul unei implementări digitale utilizând un proesor programabil. Posibilitatea de aplicare în transmiterea semnalelor de bandă largă este prezentată utilizând metoda dinamicii erorii.