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CONNECTING ANALOG AND DISCRETE NONLINEAR SYSTEMS FOR NOISE GENERATION

BY

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Abstract. Nonlinear systems exhibit complex dynamic behaviour, including quasi-periodic and chaotic. The present contribution presents a composed analogue and discrete-time structure, based on second-order nonlinear building blocks with periodic oscillatory behaviour, that can be used for complex signal generation. The chosen feedback connection of the two modules aims at obtaining a more complex nonlinear dynamic behaviour than that of the building blocks. Performing a parameter scan, it is highlighted that the resulting nonlinear system has a quasi-periodic behaviour for large ranges of parameter values. The nonlinear system attractor projections are obtained by simulation and statistical numerical results are presented, both confirming the possible use of the designed system as a noise generator.

Keywords: nonlinear systems; analogue circuits; discrete-time systems; chaotic systems; noise generators.

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1. Introduction

The properties of nonlinear systems exhibiting quasi-periodic or chaotic dynamic behavior are studied with great interest due to their useful applications. A large number of applications include true random number generation (Nejati *et al*., 2012; Callegari and Setti, 2007), noise generation (Endo and Yokota, 2007), spread spectrum modulating systems (Sambas, 2015), and chaotic encryption modules (Andreatos and Volos, 2014). Most nonlinear systems presented in the literature are either analogue or discrete. The nonlinear system generator proposed in this paper is composed of two building blocks of order two, one analogue and the other discrete leading to interesting dynamic and statistical properties.

The chosen sub-systems of the proposed nonlinear system are designed starting from Van der Pol systems (Vincent *et al*., 2014) and exhibit periodic behavior. The Van der Pol system was chosen as elementary building block because its periodic dynamics is well studied and the connection of the analogue system to the discrete version shows a dynamic and statistic behavior useful in noise generation. Due to the feedback loop connection of the analogue type of one building block and the discrete-time type of the second one, the resulting nonlinear system has a more complex behavior, for a large range of system parameters. The presented bifurcation diagrams perform a precise parametric analysis of the proposed system dynamics. Simulation results performing both dynamic and statistical analysis (Grigoras and Grigoras, 2015), confirm the possible use of the designed four-order analog-discrete nonlinear system for noise generation.

2. Nonlinear System Design

To obtain a complex dynamic behavior of the proposed nonlinear system, a connection of a discrete-time building block and an analogue counterpart is chosen. The two interconnected second-order nonlinear building blocks are designed starting from Van der Pol oscillators: one analog and the second one discrete. The state equations of the analog Van der Pol oscillator are based on the free-running frequency 'ω_a', a nonlinear term coefficient 'C_a' and an input signal $u(t)$ scaled with the *Analog Scale*, K_a, gain:

$$\begin{cases} dx_a/dt = \omega_a \cdot y_a - C_a \cdot x_a \cdot \left(1 - y_a^{\,2}\right) \\ dy_a/dt = -\omega_a \cdot x_a + u(t) \end{cases} \tag{1}$$

The index '*a*' for all equation components highlights the analog version of the analyzed building block. These equations lead to the block diagram depicted in Fig. 1.

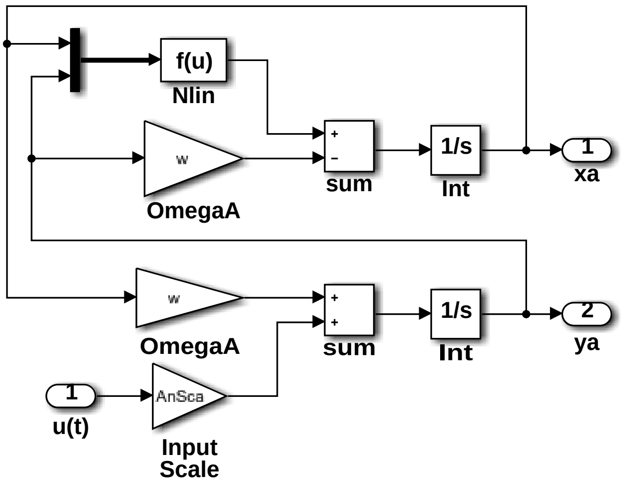

Fig. 1 – Analog Van der Pol oscillator block diagram.

The resulting oscillator generates periodic state variables, considered too simple for the desired noise generation. Thus, the analogue sub-system is connected to a similar discrete one, obtained by applying the Euler sampling method to the state Eqs. (1):

$$dx_a/dt \simeq \frac{x_d[k+1]-x_d[k]}{Step} \tag{2}$$

Applying the approximation (2) to the state Eqs. (1), we obtain the difference state equations of the discrete-time Van der Pol oscillator (3):

$$\begin{cases} x_d\left[k+1\right]=x_d\left[k\right]+Step\cdot\omega_d\cdot y_d\left[k\right]-Step\cdot C_d\cdot x_d\left[k\right]\cdot\left(1-{y_d}^2\left[k\right]\right) \\ y_d\left[k+1\right]=y_d\left[k\right]-Step\cdot\omega_d\cdot x_d\left[k\right]+e[k] \end{cases} \tag{3}$$

To avoid any confusion, similar variables and coefficients in (3) are indexed with ‘*d*’ from ‘discrete’. The block diagram of the resulting discrete system is similar to the analog variant in Fig. 1, with analog integrators replaced with discrete-time accumulators, as shown in Fig. 2. The periodic behavior of the discrete-time oscillator is highlighted by the limit cycle attractor in Fig. 3. If the discrete system receives a periodic or a random input, its behavior becomes more complex, as shown in Fig. 4 and 5, suggesting the connection with an analogue system.

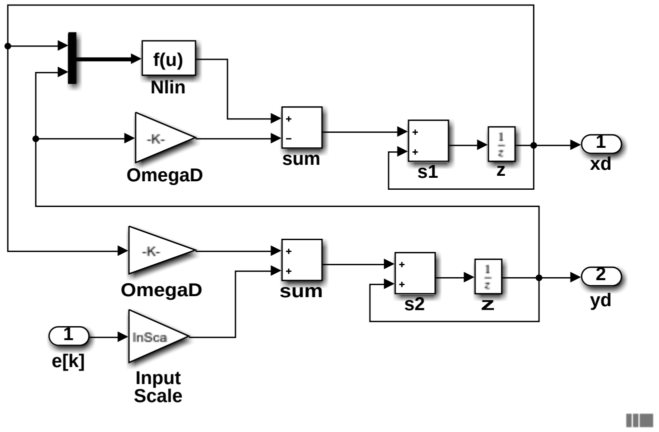

Fig. 2 – Discrete-time order two Van der Pol oscillator block diagram.

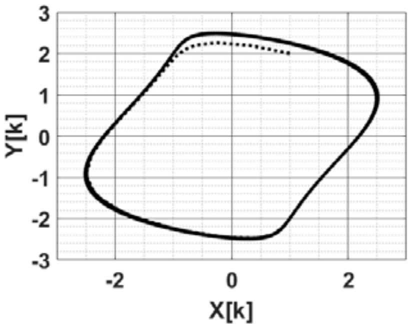

Fig. 3 – Discrete-time autonomous Van der Pol oscillator limit cycle.

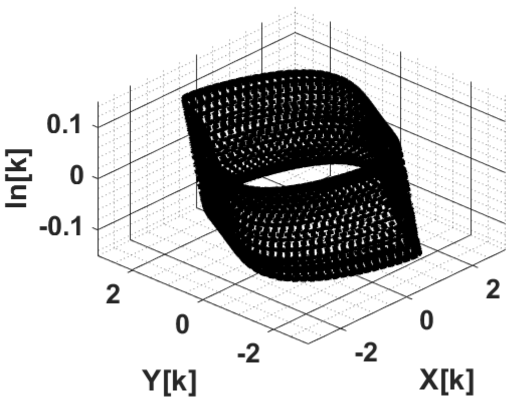

Fig. 4 – Discrete-time non-autonomous Van der Pol oscillator torus with sine input.

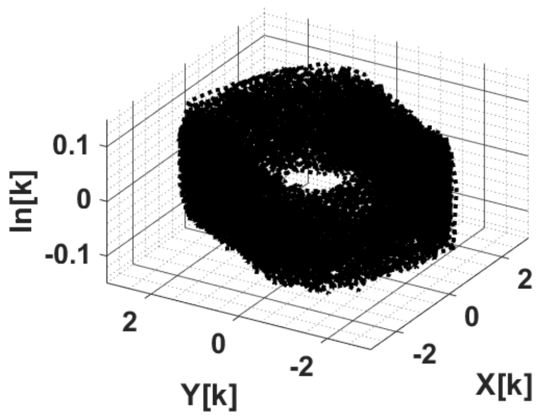

Fig. 5 – Discrete-time non-autonomous Van der Pol oscillator attractor, random input.

To obtain the desired complex nonlinear dynamic behavior, two Van der Pol order two elementary building blocks are connected in a feedback loop topology. The overall block diagram of the proposed system results as suggested in Fig. 6:

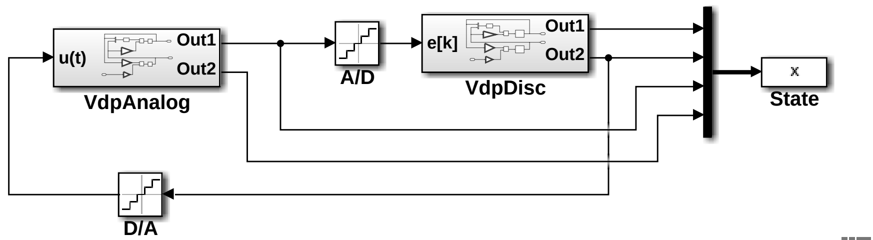

Fig. 6 – Feedback loop connection of the elementary building blocks.

Although the nonlinear characteristics of the A/D and D/A converters might influence the nonlinear dynamics of the resulting fourth-order generator system, multiple simulation tests show that the results presented in the following section are quite similar for different data converters with ten to sixteen bits precision, so conversion resolution is not further detailed.

The data conversion clock, equal to the discrete-time functioning time is denoted in the following equations as *Step*. The natural oscillation frequency of the analogue building block is denoted ω_a and that of the discrete one ω_d. Similarly, the nonlinear term coefficients of the state equations are C_a for the analogue building block and C_d for the discrete-time one and the gains for the feedback loop connection are K_a and K_d. The resulting state equations of the proposed generator system are (4):

$$\begin{cases} dx_a/dt = \omega_a \cdot y_a - C_a \cdot x_a \cdot \left(1 - y_a^{\,2}\right) \\ dy_a/dt = -\omega_a \cdot x_a + u(t) \\ x_d\left[k+1\right] = x_d\left[k\right] + Step \cdot \omega_d \cdot y_d\left[k\right] - Step \cdot C_d \cdot x_d\left[k\right] \cdot \left(1 - y_d^{\,2}\left[k\right]\right) \\ y_d\left[k+1\right] = y_d\left[k\right] - Step \cdot \omega_d \cdot x_d\left[k\right] + e[k] \end{cases} \tag{4}$$

Each input signal of a building block *u(t)* an *e[k]* are obtained by the data conversion interfaces included in Fig. 6. They are characterized by the nonlinear functions: $f_{A/D}(.)$ and $f_{D/A}(.)$, applied to a state variable from the other sub-system, as in relations (4),

$$\begin{cases} u(t) = f_{\mathrm{D/A}}(K_a \cdot y_d\,[k]) \\ \mathrm{e[k]} = f_{\mathrm{A/D}}(K_d \cdot y_a(t)) \end{cases} \tag{5}$$

3. Simulation Results

To justifiably chose the state equations coefficients, bifurcation diagrams were performed. For example, choosing ω_a = 10 and ω_d = 2.2, the discrete-time nonlinear factor coefficient, 'C_d', was slowly modified, leading to the diagram in Fig. 7. The obtained diagram highlights chaotic behavior for small values of the system coefficient, slowly changing, in the interval 0.32 – 0.36, to quasi-periodic behavior. Both nonlinear dynamic behaviors are complex enough to allow noise generation with the proposed system.

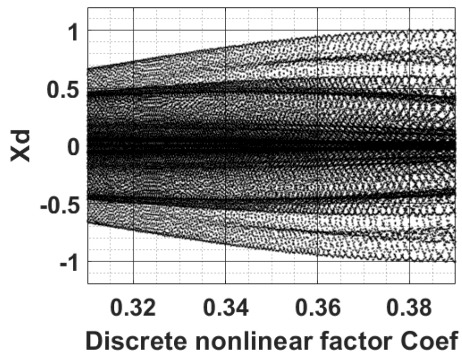

Fig. 7 – Variation of the discrete state variable x_d for Coef 'C_d' modification.

Similarly, the example bifurcation diagram depicted in Fig. 8, highlights the influence of the interconnection coefficient, In Scale K_a, in the discrete-time building block. The wide value range of the coefficient, for which chaotic behavior of the system is evident, is separated by small periodic intervals, such as 1.01, 1.06, 1.13, 1.18 and 1.24.

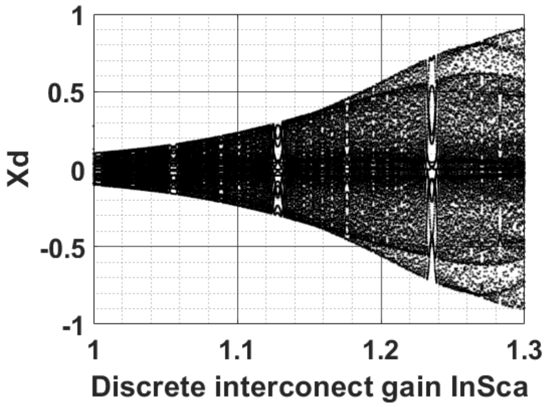

Fig. 8 – Variation of the discrete state variable $x_d[k]$ for In Scale, K_a, modification.

Similar results were obtained in the case of the parametric analysis made for the coefficients in the state equations of the analog building block.

Dynamic analysis was made for coefficient values chosen from the bifurcation diagrams, and the time evolution of the state variables in the quasi-periodic behavior is exemplified in Fig. 9. More complex graphs were obtained for coefficients leading to chaotic behavior.

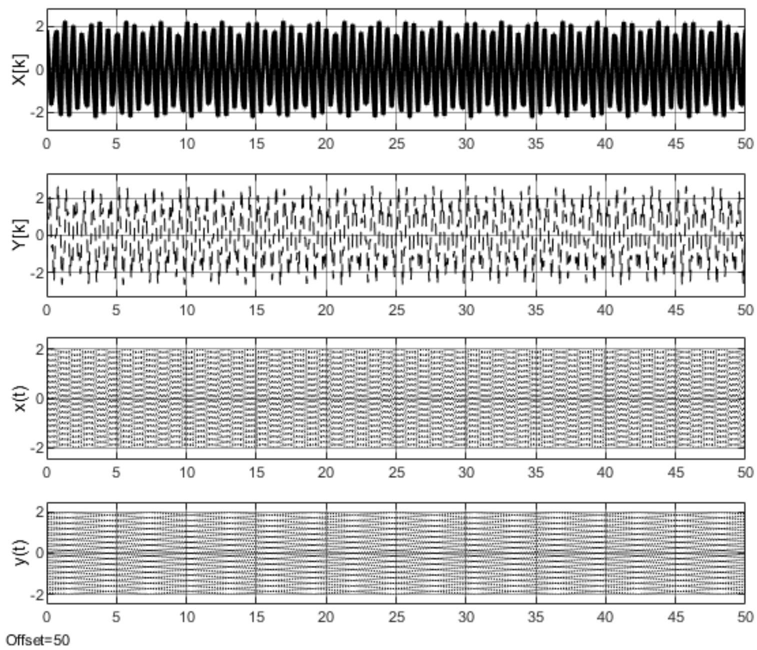

Fig. 9 – Time evolution of the system state variables.

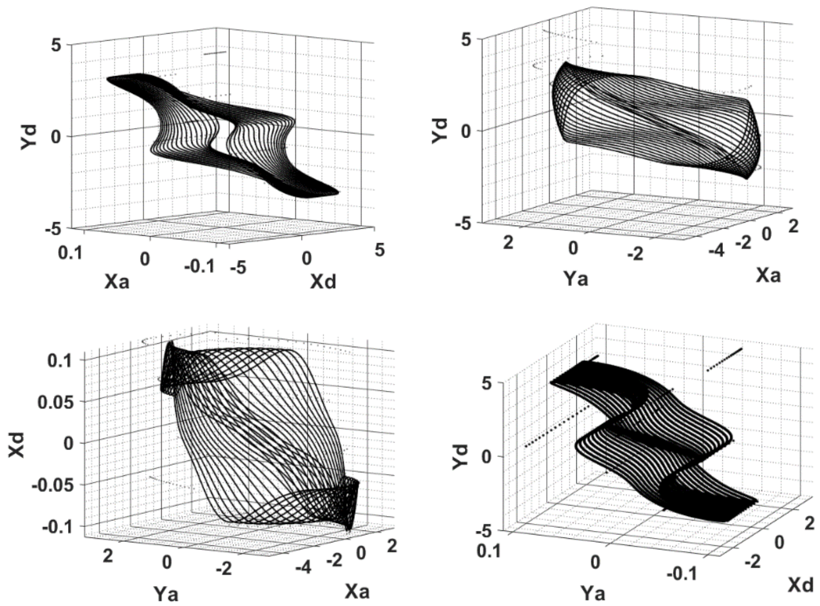

Fig. 10 – 3D projections of the system attractor for the state variables indicated on axes.

The nonlinear dynamic behavior of the compound analog-discrete system can be better seen in the graphical example of three-dimensional projections of the state attractor depicted in Fig. 10.

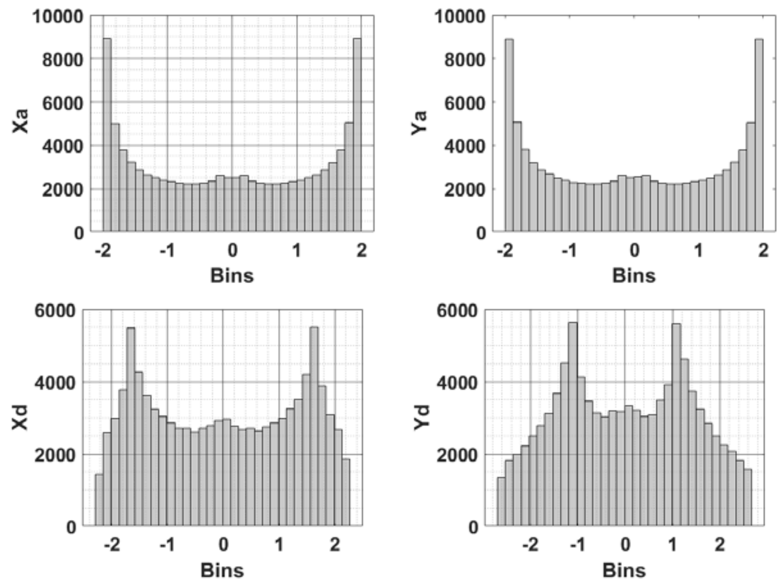

Fig. 11 – State variables histograms.

To highlight the achievement of noise generation, statistical analysis was also performed. For instance, first-order statistic results, presented in Fig. 11, show that discrete state variables, x_d and especially y_d are the best choice for approximately uniform repartition of the probability values in the case of noise generation.

Second-order statistic examples are given in the power spectrum graphs depicted in Fig. 12 for the discrete state variables, x_d and y_d. The approximately uniform low-frequency power repartition, in the zero to 10^{-1} frequency band, is useful and suggests a narrow time extension of the state variables autocorrelation function.

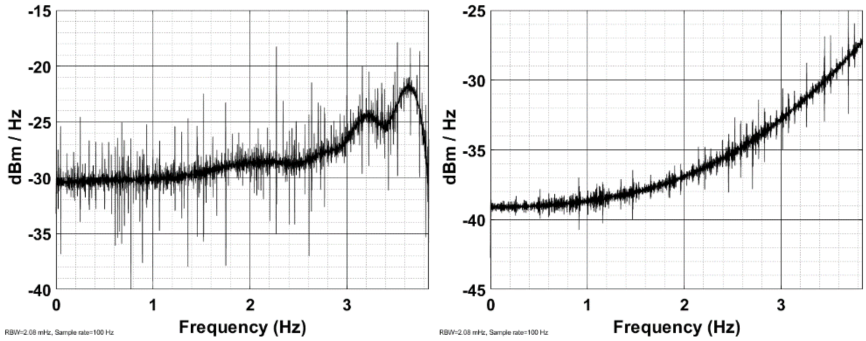

Fig. 12 – Xd and Yd power spectra.

4. Conclusions

We have developed an order four nonlinear system aiming at applications of noise generation. The proposed design is based on the feedback interconnection of two periodic oscillators. To obtain a more unpredictable behavior of the resulting system, the two basic blocks are dynamically different: one analog and the other discrete-time. By performing several bifurcation diagrams, we highlighted the complex behavior of the designed system for a wide range of parameter values. Simulation performed on the nonlinear system for several sets of parameters confirmed its complex behavior and the statistical results suggest it can be used as a noise generator.

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CONECTAREA SISTEMELOR NELINIARE ANALOGICE ȘI DISCRETE PENTRU GENERAREA ZGOMOTULUI

(Rezumat)

Sistemele neliniare prezintă un comportament dinamic complex, inclusiv cvasi-periodic și haotic. Această lucrarea prezintă o structură compusă analogică şi discretă, bazată pe blocuri neliniare de ordinul doi, cu un comportament oscilatoriu periodic, care poate fi utilizată pentru generarea de semnale complexe. Conexiunea cu reacție a celor două module are ca scop obținerea unui comportament dinamic neliniar mai complex decât cel al blocurilor constructive. Efectuând o scanare a parametrilor se evidențiază faptul că sistemul neliniar rezultat are un comportament cvasi-periodic pentru intervale mari de valori ale parametrilor. Proiecțiile atractorului neliniar al sistemului sunt obținute prin simulare și sunt prezentate rezultate numerice statistice, ambele confirmând posibila utilizare a sistemului proiectat ca generator de zgomot.