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ANALOGOUS MODE IMPLEMENTATION OF STATE SPACE BASED CONTROL OF BUCK CONVERTER

BY

DIANA-ELENA CUCOŞ* and DORIN O. NEACŞU

“Gheorghe Asachi” Technical University of Iaşi,
Electronics Department

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Abstract. The new requirements for point-of-load converters include the dynamic behavior, leading to the need for improvements in the control system for the conventional buck converter. Usually, the regulator design, in order to secure stability at parameter or mode variation, aims a zero steady-state error and a phase margin over 60 degrees. The design based on Venable’s K-factor method (Venable, 1983) helps achieve this topic. A more modern control design method based on State Space is used progressively on digital platforms. This paper extends the merits of the State Space based design procedure with an analogous mode implementation.

Keywords: dc/dc converters; feedback control; State Space based control; operational amplifier.

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*Corresponding author; *e-mail*: diana-elena.cucos@student.tuiasi.ro

1. Introduction

Control of point-of-load converters (Neacşu, 2017) - used to power low-voltage loads- is primarily implemented on platforms built using integrated circuits (ICs). These integrated circuits follow Mammano's power IC designs (Mammano, 2007) from the 1970s.

Common applications for these switch mode power supplies include regulator functions where the load voltage must be kept constant at a specific reference value (Ghosh *et al*., 2014; Franklin *et al*., 2014; White, 2016). This means that their design requirements imply stability with large enough margins and zero steady-stateerror (Priewasser, 2012; Poodeh *et al*., 2007; Olalla *et al*., 2010). At the other end of the design spectrum, state-space-based control emerges as a purely digital solution (Marsala and Ragusa, 2012). This is also called modern control because modern people use computers for design and implementation (Modabbernia, 2013). In a state-space-based control design approach, the dynamic requirements are translated into desired locations of system poles (Neacşu, 2016a; Neacşu, 2016b). With the use of a linear control law that is generated in MATLAB® using the "acker" or "place" commands (Neacşu and Şirbu, 2018a; Neacşu and Şirbu, 2018b), the existing poles from the buck converter model are transferred to the new location. This straightforward but very mathematical method ensures dynamic performance.

This paper extends the merits of the State Space based design procedure with an analogous mode implementation.

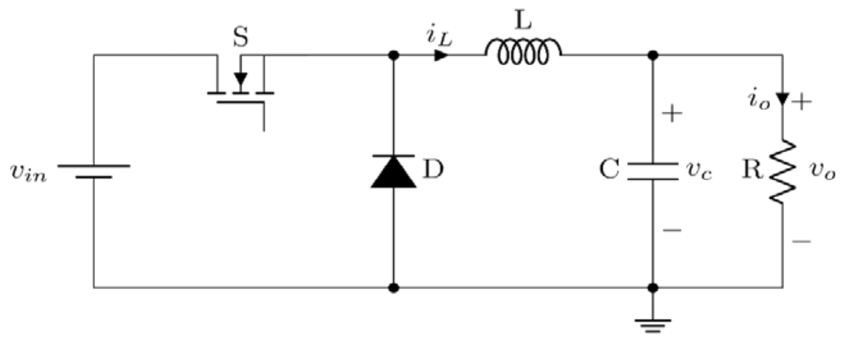

Fig. 1 ─ Buck Converter.

2. State Space Based Control

A. Theory

The equations are represented in state space as part of a contemporary control approach. The results of the State Space equations for a buck converter are:

$$\begin{cases} \dot{X} = \begin{bmatrix} 0 & -\frac{1}{L} \\ \frac{1}{C} & -\frac{1}{R \cdot C} \end{bmatrix} \cdot X + \overbrace{\begin{bmatrix} \frac{V_{IN}}{L} \\ 0 \end{bmatrix} \cdot d}^{\text{input}} + \overbrace{\begin{bmatrix} \frac{D}{L} \\ 0 \end{bmatrix} \cdot v_{IN}}^{\text{Perturbation =0, when Vin=ct}} \\ Y = [0 \quad 1] \cdot X \end{cases} \tag{1}$$

We have the following state variables: the input voltage (v_{in}) and load current (i_{load}), the control manipulator variable is duty cycle (d), the inductor current (x_1=i_L) and capacitor voltage (x_2=v_c). These variables are all examples of small-signal elements. The boost inductance is represented by *L*, the output capacitor is represented by *C*, the load resistance is represented by *R*, and the bias (nominal) duty cycle is represented by *D* in the matrix coefficients.

System pole displacement into another location is the goal of the State Space-based Control Law, which contributes to improve the transient response and stability (Abbas *et al.*, 2010). A steady-state error would result from such a control system, which would be affected by parameter variation. Adding a new state variable as the error integral is the solution that can reduce the steady-state error.

Because it tends to slow down the transient response, adding an integral term to the converter poles has a negative impact on the dynamic behavior (Kelly and Rinne, 2005). A feed-forward term [N] from reference to the plant input is added to account for this. This [N] gain was chosen such that it would result in a zero at the same frequency with the pole originating from the integrator in its new location.

Hence:

$$u = -K \cdot [X] = -[K_1 \quad K_2 \ldots \quad K_n] \cdot \begin{bmatrix} x_1 \\ x_2 \\ \cdots \\ x_n \end{bmatrix} \tag{2}$$

$$N = -\frac{K(gain\ for\ integrator)}{P(new\ location\ of\ the\ pole\ derived\ from\ integrator)} \tag{3}$$

In order to demonstrate an equivalent to the more conventional PI control, the control system can be rearranged in the manner depicted in Fig. 2. It is possible to estimate or measure the inductor current.

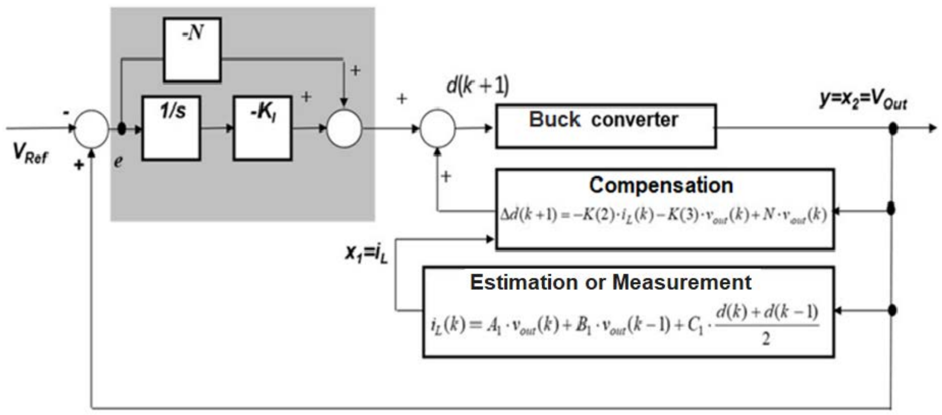

Fig. 2 – PI equivalent form of the state-space controller.

B. MATLAB® Based Design

The control gains are calculated in MATLAB with instruction “acker” or “place” based on a desired location of poles. A code example is next included.

```
% State space equations for Buck converter
F=[0 -1/L; 1/C -1/(R*C)];
G=[Vin/L; 0];
Gw=[D/L];
H=[0 1];
%--------------
% New desired locations (non-optimal design)
% - select a frequency  f=5kHz-> 2πf=wn=31416 rad/sec,
% - select a damping ζ=0.6 for an overshoot of Mp=10%,
% and phase margin of PM=60 degree
% Desired location of poles is Pd
fnn=5000; wnn=2*pi*fnn;
zz=0.6;
xreal=zz*wnn; ximag=wnn*sqrt(1-zz^2);
Pd=[-xreal+ximag*i -xreal-ximag*i -wnn];
%--------------
% State matrix, including the integrative term is rebuilt
% to show 3 state variables
%The state equations from the initial matrices
Fp=[0 H; zeros(size(F,1),1) F];
Gp=[0; G];
```

```
Hp=[0 H];
Jp=0;
%--------------
% Calculate K gains using acker/place
K1=acker(Fp,Gp,Pd)
```

A new location of poles for a closed-loop system is usually determined based on dynamic requirements for the final system. In most cases, the dynamic requirements are defined based on a step response.

The pole placement is determined by the following calculation:

- Rise time $T_r < 75$ μs => $\omega_n > 24000$ rad/s
    - Adopt 31416 rad/s = 5 kHz
        - bandwidth of $1.2 \cdot \omega_n = 37700$ rad/s
- Overshoot < 10%
    - Equivalent damping $\zeta > 0.6$
        - Phase margin PM > 60
- Stabilization time: < 300μs
    Real part of poles < -4.6/300us = -15.3 krad/s
    - Consider -18850 rad/sec.

Consequently, we adopt

- two poles at –18850 +/– j·25132 [rad/sec];
- one pole at –31416 [rad/sec] for the error integrator.
- one zero at –31416 [rad/sec] for the feed forward term, able to cancel the pole resulted from error integrator usage.

Using MATLAB®, the following control gains yield:

$$K1= [8009.8 \quad 0.1 \quad 0.4] \tag{4}$$

$$N= 0.2550 \tag{5}$$

This procedure is schematically shown in Fig. 3, after the gains are grouped like in Fig. 2 with separate PI and a compensation term.

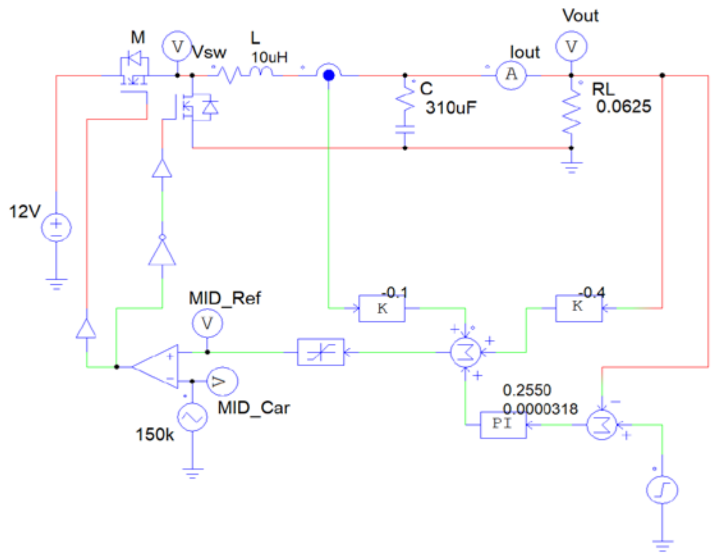

Fig. 3 – Circuit equivalent for the State Space implementation.

3. Implementation with Operational Amplifiers

A. Current Measurement

Figure 4 shows the circuit for measurement of the inductor current. It is based on a measurement bridge including the boost inductor and an R-C network. The voltage drop on the capacitor equals the inductor current if the product of resistance R and capacitance C equals ratio between inductance and its parasitic resistance. This voltage is amplified according to the gain designed in the previous section.

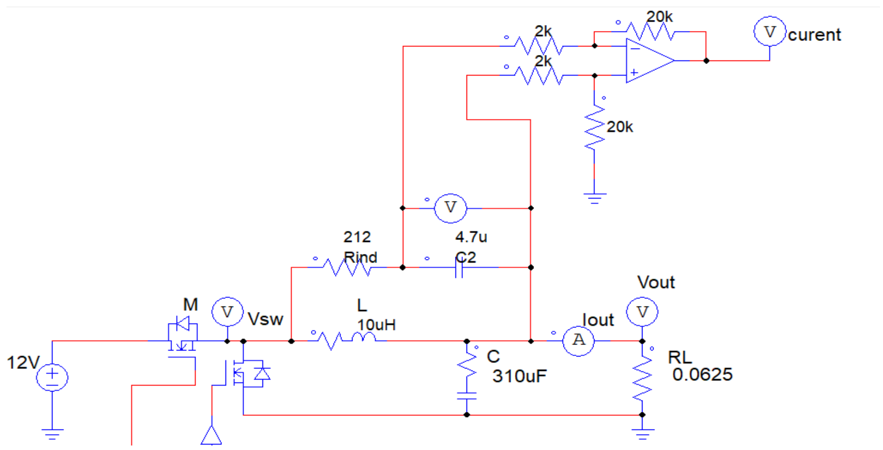

Fig. 4 – Current measurement.

B. Voltage Measurement

Figure 5 shows the circuit for measurement of voltage, using the inverting operational amplifier.

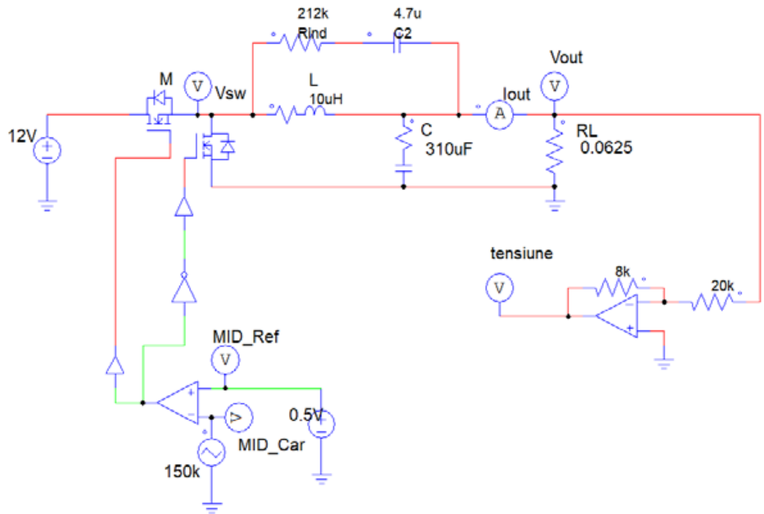

Fig. 5 ─ Voltage measurement.

C. PI Control

The Proportional-Integral (PI) Controller has the output limited to the upper and lower limits. This PI controller is implemented using an operational amplifier:

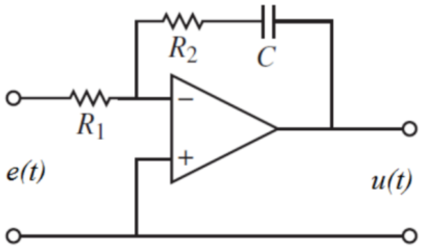

Fig. 6 ─ Operational Amplifier.

In Fig. 6:

- this operational behaves like a PI controller when a capacitor is added in series with one of the resistors
- the capacitor is used to store charge and represents the integral of the input
- Proportional gain: $Kp = R2/R1$ (6)
- Integral time: $Ti = R2*C2$ (7)

D. Final Circuit

Figure 7 represents the final circuit composed of the chosen amplifiers.

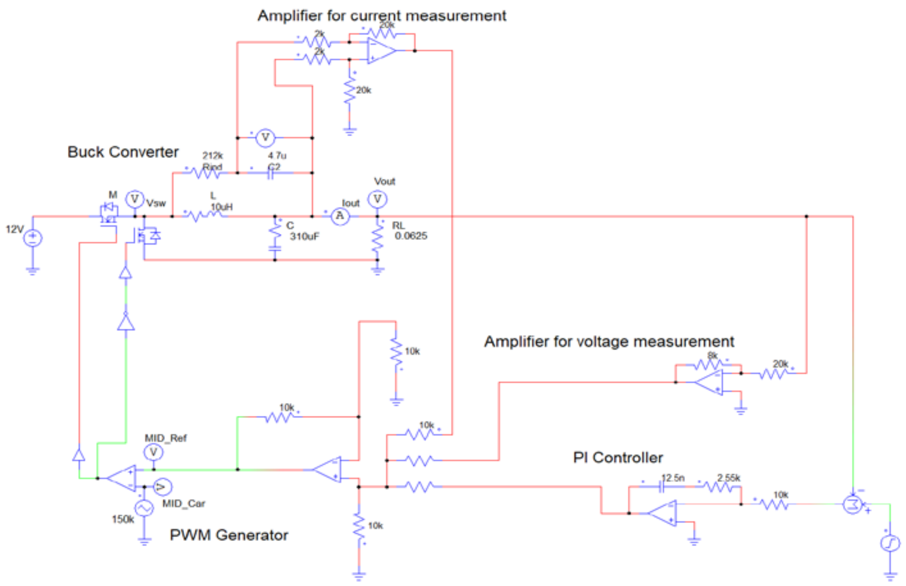

Fig. 7 – Final circuit.

4. Results

The proposed implementation method is verified in PSIM.

Table 1
Values for the elements of the amplifiers

Amplifier for current measurement	Amplifier for voltage measurement	PI Controller
R1 = 20 k	R5 = 8 k	R9 = 10 k
R2 = 2 k	R6 = 20 k	R10 = 2.55 k
R3 = 2 k		C1 = 12.5 n
R13 = 20 k		

Figure 8 illustrates a step response where the reference for the output voltage is changed from 3.3V to 5.0V. For completeness, Fig. 9 illustrates details of the waveforms in steady state.

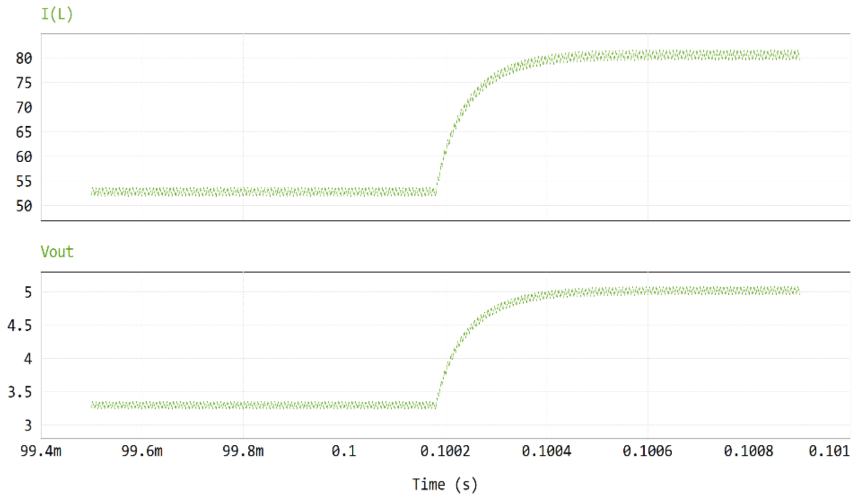

Fig. 8 ─ Step up change in the output voltage for the simulated circuit.

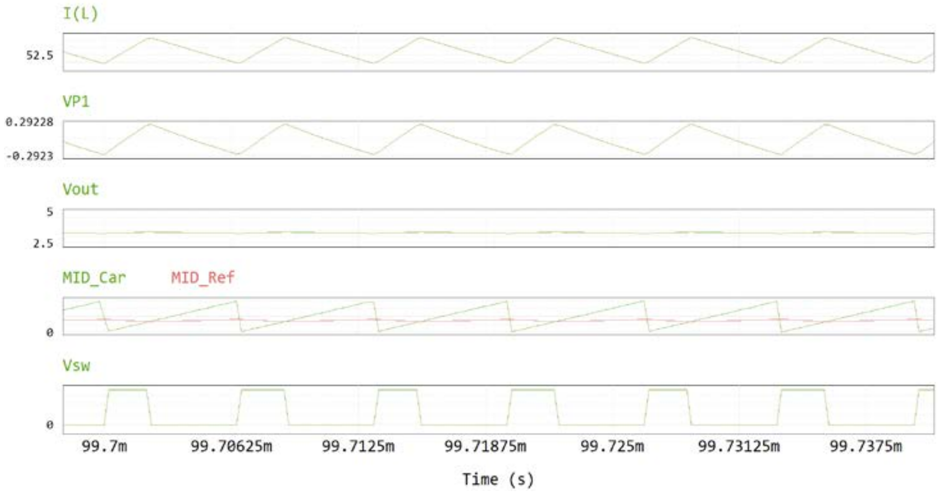

Fig. 9 ─ Waveforms for the operation of the circuit (from top to bottom): inductor current, sensed voltage proportional to the inductor current, output voltage, PWM carrier and reference waveforms, voltage across diode in the Buck converter.

5. Conclusion

The development of computer and telecommunications applications has raised the power supply requirements, which now include particular dynamic requirements.

This paper considers the State Space based control method, which can be used for the control of point-of-load converters. The method utilizes the computer design for the State Space based control. While this method is entirely dedicated to a digital implementation, this paper forces an analog implementation with power IC (integrated circuits) devices in order to offer some grounds for comparison. The State-Space based method starts the design from choice of pole location for the final closed loop system. In this respect, design requirements are equivalently mapped into a complex plane with an equivalence to a 2-pole system.

Controllability and compliance with the design specifications are guaranteed by a straightforward State Space design procedure in MATLAB®. Additionally, this paper demonstrates how to adapt the digital State Space-based design approach to an analog implementation platform.

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IMPLEMENTARE ÎN MOD ANALOG A ECUAȚIILOR DE STARE PENTRU UN CONVERTOR BUCK

(Rezumat)

Noile cerințe pentru convertoarele de tip point-of-load includ comportamentul dinamic, iar acest lucru necesită îmbunătățiri ale sistemului de control pentru convertorul buck convențional. În mod tradițional, proiectarea regulatorului urmărea o eroare zero steady state și o margine de fază de peste 60 de grade, pentru a asigura stabilitatea la variația parametrilor. Acestea au fost obținute folosind o proiectare bazată pe metoda K-factor a lui Venable.

O metodă de proiectare de control modernă, bazată pe Ecuațiile de Stare, este folosită din ce în ce mai mult pe platformele digitale. Această lucrare extinde meritele acestei metode, folosind o implementare în mod analog.